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CONTENTS

Hydrology. — *How can intermittence of springs be explained?* By J. VERSLUYS.

(Communicated at the meeting of February 23, 1929).

A mixture of gas or vapour and liquid in a subterranean channel can rise regularly under two conditions, called the foam and the mist conditions. Which of these will arise depends on the proportion in which gas and liquid are mixed. This proportion is not only determined by the proportion in which they flow out but also by their difference of speed and their absolute velocity. Due to the fact that difference of speed is not the same under both conditions, direct transition from one condition to the other is impossible. If in a part of the channel the intermediate conditions prevail intermittant flow will be the result.

Irregularity of flow frequently occurs with hot and gaseous springs, and in many cases this irregular flow passes into intermittence.

No satisfactory explanation of this phenomenon has yet been given. In the middle of last century intermittence of springs was explained by the assumption that the channel through which the liquid is conveyed to the surface had the shape of a syphon. (II). This assumption is not very tenable and it does not allow for the presence of gas and vapours.

L. DE LAUNAY's theory (III) concerning the intermittence of hot springs is, it is true, less artificial, but it cannot be accepted in general because of its being based on the supposition that there are domeshaped enlargements in the crevice through which the liquid and gas rise to the surface. Such a dome might disturb the regular flow of the spring in the following manner:

Let us assume for a moment that the dome is filled with water. Then the rising gas will collect in this space, and as long as this takes place the column of liquid in the crevice from the dome upward is filled with water only. After the dome is entirely filled with gas, however, the gas will pass this dome and rise to the surface, reducing the weight of the liquid column above the dome. The gas in the dome will then expand and be partly discharged, still further diminishing the specific gravity of the mixture of water and gas in the crevice from the dome to the surface. Still more gas will escape, but eventually the gravity of the mixture will increase again when the gas in the dome is compressed and the gas coming from below will again collect therein. A dome shaped enlargement of the crevice may cause intermittence, but, as intermittence and irregular flow are often observed with oil wells producing a mixture of oil and gas through a straight string without contractions, there must be another and more general cause

of irregular flowing and intermittence with mixture of liquid and gas rising through a vertical or inclined channel.

Generally hot springs originate from crevices, establishing a connection between deep-seated strata and the surface. Such crevices are filled with water. The pressure of the water occluded in the pores of the earth layers increases according to the depth, sensibly at the same rate as the hydrostatic pressure ; thus pressure increases one atmosphere at about ten metres depth.

Where normal conditions prevail temperature increases 1 degree centigrade at about 30 to 35 meters' depth, and due to this increase of temperature vapour pressure also increases with the depth, though at a smaller rate than hydrostatic pressure. Therefore no evaporation can take place in deep seated strata, nor in crevices under normal conditions. Only in volcanic regions may vapour be generated in deeper strata.

Gas-bearing sediments and volcanic rocks may cause water to rise in crevices due to the gas entering therein. But also artesian pressure may establish the motion. Once motion is started in a crevice, water with a higher temperature is lifted to shallower depths where smaller pressures prevail and vapour may form. As soon as these conditions arise vapour promotes the flow by reducing the weight of the liquid column in the channel.

The fact that irregular flow and even intermittence are so often observed with hot and gaseous springs, whilst the same occurs in straight tubes of oil wells, may probably be explained by the following considerations.

In the mixture of gas and liquid, gas may be dispersed in the shape of bubbles in the liquid, but also the liquid may be dispersed in the gas in the shape of drops. The two conditions here described will be called the "foam" and the "mist" conditions.

Under both conditions in a vertical or inclined channel gas rises at a greater speed than does liquid (see V). Suppose the proportion of volumina in which gas and liquid pass a cross-section to be $n:1$, i.e. every unit of volume of liquid is accompanied by n of the same units of gas, then if the gas has a velocity of u, and the liquid one of $u-b$, the unit of volume of liquid flowing through in a unit of time occupies an area

$$\frac{1}{u-b} \quad \ldots\ldots\ldots\ldots \quad (1)$$

in the cross section, and the n units of volume of gas, accompanying this liquid occupy an area of

$$\frac{n}{u} \quad \ldots\ldots\ldots\ldots \quad (2)$$

Thus the area of the cross section of the channel per unit of volume of liquid flowing through per second is :

$$S=\frac{n}{u}+\frac{1}{u-b} \quad \ldots\ldots\ldots \quad (3)$$

From (1) and (2) it follows that gas and liquid are mixed in the proportion

$$\varphi = \frac{n}{u} : \frac{1}{u-b} = n : \frac{u}{u-b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

or

$$\varphi = \frac{n\,(u-b)}{u} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

In (4) being $\frac{u}{u-b} > 1$ the proportion $\frac{gas}{liquid}$ in the mixture is always smaller than the proportion n in which gas and liquid flow through.

Eliminating u from (3) and (5) we obtain the following equation

$$S = \frac{(n-\varphi)(\varphi+1)}{\varphi b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Substitution of the values of n and b in this equation leads to a relation between S and φ.

As stated before two values of b are possible. The velocity of a gas bubble rising in water, which is at rest, is about 20 and 30 cm per second and the velocity of small drops of water sinking or falling in a gas may be about 600 cm per second. Although in a rising mixture the values of b are not constant, for sake of simplicity it will be taken that under foam condition

$$b = 20 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and under mist condition

$$b = 600 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

centimeters per second.

As an example, in figure 1 there are plotted in semilogarithmic coordinates the percentages of gas in the mixture against the values of S for $n = 6$. In this diagram the values of S are expressed in square centimeters per cubic centimeter of liquid flowing through per second.

The values of S have been computed by the aid of (6) in which have been substituted the values of φ corresponding to the different percentages of gas in the mixture.

On account of the existence of two values of b, see (7) and (8), two lines can be traced.

If, now, we assume that foam condition in a mixture is able to exist as long as less than 50 % gas is present, and that with more gas the mist condition would arise which is probably not exactly the case, then only the traced parts, of the two lines would be available, the dotted parts not corresponding to real conditions. Then, although generally speaking with 50 % gas in the mixture, there would be a transition from foam into mist, in the particular case of the mixture rising in a channel no direct transition from foam to mist can be established by gradually decreasing the section. The proportion of 50 % has been chosen, because as soon as nearly 50 %

of the mixture is gas, the bubbles will touch each other and under these circumstances a non stabilized foam will break.

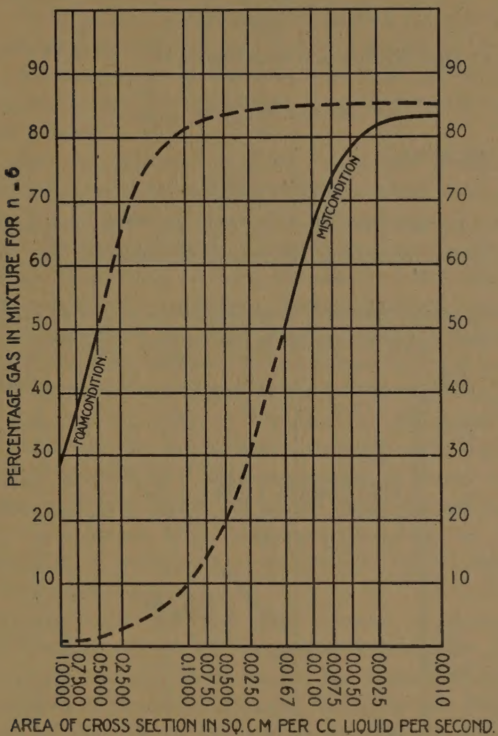

Fig. 1.

Within the range of values of S between the limits 0.5000 and 0.0167, none of the conditions will exist. These figures, read from the diagram, are only available if $n = 6$, but for any arbitrary value of n, a similar diagram can be drawn and a range of values of S will be found to exist, which do not allow the mixture to rise either under foam or under mist condition. Within the range of these values of S the mixture is not able to flow regularly.

According to the assumption that 50 % gas, i.e. equal volumes of gas and liquid in the mixture, would be the limit at which the transition from foam to mist may occur, in (6) can be substituted

$$\varphi = 1 \quad \ldots \ldots \ldots \ldots \quad (9)$$

in order to obtain an equation involving, n and the limit S_l of S:

$$S_l = 2\,\frac{n-1}{b} \quad \ldots \ldots \ldots \ldots \quad (10)$$

The expression of the limit of foam condition would be in accordance with (7)

$$S_{lf} = 2\frac{n-1}{20} = \frac{n-1}{10} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and the limit of mist condition is constituted by :

$$S_{lm} = 2\frac{n-1}{600} = \frac{n-1}{300} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Thus one can write

$$S_{lf} = 30\, S_{lm} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In diagram 2 the lines representing (11) and (12) are traced, the limits S_{lf} and S_{lm} being computed in square centimetres per litre of liquid per second.

This diagram shows that only large quantities of gas or vapour or narrow channels are favourable to the existence of mist condition.

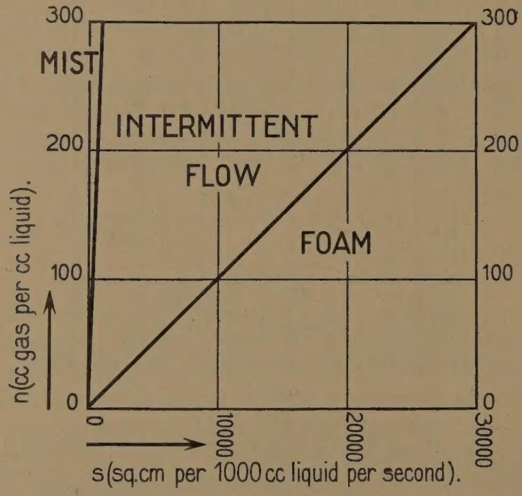

Fig. 2.

Suppose the crevice of the spring has a unform cross section from the bottom to the surface, so that the magnitude S in the above formulae remains constant. But as the mixture rises the pressure decreases, and accordingly the volume of gas (n in the formulae) increases. Suppose the value of S is 10000, and the value of n at the bottom of the channel is about 6, then diagram 2 shows that at the bottom of the well the foam conditions prevail. Following the vertical line 10.000 in the diagram, we see that as the value of n has increased to about 101 the limit if the field of intermittent flow has been reached. What will take place here ? We can imagine that the mixture rises under the foam condition to the cross section where pressure has diminished so far as to make $n = 101$. From this section

upwards no foam condition can exist, but the volume of gas is not sufficient to entrain all the liquid in the shape of drops, and in practically all cases the velocity would be smaller than 600 cm per second (see 8) so that no liquid will be lifted at all. Therefore generally all the liquid, and under exceptional circumstances only a part of it will remain at the level of the cross section considered here. In this way more liquid will collect here and for a certain lapse of time less liquid than is yielded by the formation will flow out of the channel.

This will create an unstable condition, as it also interferes with the pressure in the lower parts of the crevice and the result will be an irregular flow and even, if conditions are favourable, intermittence.

In arriving at the above deductions figures have been taken for the rate of fall of the drops of liquid and for the rate at which the gas bubbles rise, but the figures taken here do not always hold good, and they depend upon the circumstances as has been stated above. Therefore the above deduction is only to be regarded as a demonstration of the basic principle.

Probably the limits of the range of values of S causing intermittent flow are not very sharp. If the value of S is near the middle of the range of intermittent flow, then the spring will alternately eject liquid occluding little gas and gas containing a small percentage of liquid in drops. The nearer the value of S approaches one of the limits, the shorter the period of intermittance will become and at last, but still before the critical values have been reached, one can speak of a regular flow, though the liquid forms large, irregularly shaped bodies.

Similar phenomena are observed with volcanos, so that probably the same principles apply to volcanic eruptions. In the case of volcanism however, the specific gravity of the liquid, the magma, is about $2\frac{1}{2}$ and accordingly under mist condition the value of b must also be greater. Due to the viscosity of the magma under foam condition, b is probably very small. Further one may conclude from the fact that certain volcanic material floats on water, that values of φ greater than 1 may exist under foam condition.

An example of a viscous liquid rising under foam condition is presented by the pitch lake of Trinidad.

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Geophysics. — *A gravity expedition of the U. S. Navy.* By F. A. VENING MEINESZ.

(Communicated at the meeting of February 23, 1929).

In the course of 1928 the writer received an invitation from the Carnegie Institution of Washington to come to the U. S. A. with the apparatus for maritime gravity survey of the Netherlands Geodetic Commission in connection with an expedition for determining gravity at sea, which the U. S. Navy wished to organize as a consequence of a communication on this subject, received from the International Geodetical and Geophysical Union. The Secretary of the Navy designated the Naval Observatory for making the arrangements for this expedition.

The expedition took place in the autumn and consisted of the U. S. Submarine S-21 on board of which the observations were made and two surface ships, the U. S. Eagles 35 and 58. The captain of the Eagle 35, Lieutenant T. L. NASH had command of the expedition, while Lieutenant JAMES L. FISHER was in command of the U. S. S. S-21 ; the officers of this ship were Lieuts. F. S. HALL, F. DOW HAMBLIN and A. R. SODERGREN. The scientific staff on board the S-21 consisted of Dr. FRED. E. WRIGHT of the Geophysical Laboratory of the Carnegie Institution, Mr. ELMER B. COLLINS, principal Scientist of the Hydrographic Office, and the writer.

The observations have been made with the pendulum apparatus which has been constructed according to the plans of the writer at the laboratory of the Meteorological Institute at De Bilt (Holland) by the chief mechanic L. M. VAN REST ; begun in 1925 it has received its final shape in the spring of 1928 by the rearrangement of the photographic recording apparatus and other smaller improvements.

The apparatus was mounted in the central control room of the submarine. As it was near the meta-centre of the ship, the rolling and pitching of the ship caused hardly any translations of the apparatus, which is of course advantageous. It is hung in gimbals so that it is possible to work even if the rolling or pitching is fairly large ; during the last expedition, a few observations have been made with a roll of 7° to both sides of the vertical and with a slight modification of the gimbals it is hoped that in the future even greater movements may be allowed. In this way it is often possible to make the observations at periscope depth. During the return, however, from Guantanamo across the Atlantic to Washington, the sea was so rough, that two observations, which had been planned above the bottom and the top of the continental slope, could not be made ; the movements at 30 meters depth far exceeded the limit of 7°.

The schedule of the expedition has been arranged according to the different geodetical and geophysical problems in or near the West Indies. We may mention the question if the Gulf of Mexico and the Caribbean Sea are in isostatic equilibrium, if the Mississippi delta shows deviations of isostasy, of the gravity field above the Nares deep North of Porto-Rico and the Bartlett deep South of East Cuba, of the gravity in the Atlantic between Cape Hatteras and the West Indies and especially above the continental slope for that part of the coast of the U. S. The expedition has accomplished the following programme :

Leave	Arrive	Number of Observ.
1 Oct. Washington.	2 Oct. Hampton Roads.	0
4 „ Hampton Roads.	8 „ Key West.	3
10 „ Key West.	14 „ Galveston.	7
19 „ Galveston.	23 „ Key West via Mississippi delta.	7
29 „ Key West.	2 Nov. Guantanamo (Cuba) via Bartlett Deep.	8
5 Nov. Guantanamo.	9 „ St. Thomas via Nares Deep.	9
15 St. Thomas.	19 „ Guantanamo via Caribbean Sea.	6
21 „ Guantanamo.	27 „ Washington.	5

Besides, observations have been made in all the harbours, of which Hampton Roads, Guantanamo, San Juan (during trip from Guantanamo to St. Thomas) and St. Thomas were gravity stations, which have not yet been occupied before. The total number of new stations amounted therefore to 49.

Thanks to the whole-hearted cooperation of the Captain of the U.S.S. S-21, Lieut. FISHER, who ordered all the dives necessary for the observations, the possibilities given by this programme could be realized completely. This implied a great deal of diving that is to say an additional strain for everybody ; often several submergencies were made on one day, once even five in seventeen hours. The helpful assistance of the Captain, the Officers and the crew under all circumstances may be gratefully acknowledged here.

Besides the pendulum observations a great number of soundings has been taken. The submarine was provided with the sonic depth-finder of the U. S. Navy, which has given good results. Not only were soundings possible during submergence but also while going at the surface ; only when the sea was very rough the echo could not be observed. In this way a continuous line of soundings could be taken over the whole route of the expedition with the exception of a few rough days in the Atlantic during

the return voyage. Over the deeps the soundings were taken at short intervals, so that a detailed profile could be obtained ; no instances have been met, as has been the case during the previous voyage with the K XIII over the West Pacific deeps, of irregular and disappearing echo's. To the sides of these deeps, steep slopes have been found — south of East Cuba an inclination of 40° — but, with only few exceptions, the bottom of the deeps appeared to be fairly even.

The results of the gravity observations have been provisionally computed during the voyage itself and at the same time the isostatic reduction of the stations has been made at the bureau of the U. S. Coast and Geodetic Survey, so that the provisional results were already available a few days after the return of the expedition to Washington. The final computations are now being made at the Naval Observatory. As long as they have not been finished, a definite interpretation seems not advisable but a few remarks concerning the provisional results may be given ; it is not probable that the final results will change these conclusions [1]).

The Gulf of Mexico shows a positive anomaly of about 60 millidynes over nearly its whole extension ; only North of Yucatan and South of the Mississippi delta the anomaly is less, but it has still a positive value there of about + 20 m.d. It is not easy to see the significance of this positive anomaly because it seems difficult to accept the existence of stresses in the earth's crust working over so large an area and not showing any evidence of their presence in the adjoining land areas. Have we to look here for a deep-seated cause, connected perhaps to the origin of the Gulf ?

We may recall here to mind that previous observations which have been made in another inland sea, i.e. in the Mediterranean, likewise have given positive anomalies which locally even attained values of about 100 m.d. For the Western part of the Mediterranean, KOSSMAT has proposed the following explanation [2]). The surrounding mountain-chains have sunk after folding, according to the readjustment of isostatic equilibrium and the magma has been pressed away towards the region below the sea, causing there an excess of mass, which has not yet sunk into isostatic equilibrium. The writer has some difficulty in accepting this explanation. Letting alone the question if indeed the pressing away of the magma would cause so much horizontal movements in the subcrustal magma, which seems doubtful to him, he has the objection that, according to this view, the mountain chains ought to show still greater positive anomalies after isostatic reduction than the sea between and that this is not the case in general.

Returning to the Mexican Gulf, we may mention that in the region near the Mississippi delta, where we might expect a positive anomaly because of the large masses of material deposited by the river in the gulf, no evidence

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1) All the anomalies mentioned in this paper refer to gravity values, which have been reduced isostatically according to the method of the U. S. Coast and Geodetic Survey.

2) FR. KOSSMAT, Die mediterranen Kettengebirge in ihrer Beziehung zum Gleichgewichtszustande der Erdrinde.

of such an effect has been found; the small anomalies seem rather to be connected to the general trend of anomalies in the whole gulf. We may conclude therefore that there is no evidence of a lag in isostatic adjustment of the delta.

The Caribbean Sea shows also positive anomalies for the part which has been investigated, i.e. the part South of Haiti and Porto-Rico. The mean of the four available gravity anomalies after isostatic reduction is + 40 m.d.

Above the Nares deep, North of Porto-Rico, great departures of isostatic equilibrium have been found, which is not surprising as it may be considered probable that these ocean deeps are not yet in stable condition. Above the deep itself is a great negative anomaly; before isostatic reduction the deficiency is more than 300 m.d. and after this reduction it is still more than 190 m.d. This result is in good agreement with the few gravity values which have been determined in this region during the previous voyage with Hr. Ms. K XIII. North and South of the deep, the anomaly after isostatic reduction is small; to the North it is slightly positive, ± +25 millidynes, and to the South, in the harbour of San Juan, slightly negative, —10 m.d. The gravity on the island of Porto-Rico is yet unknown, but in a short time some data will be available; the U. S. Coast and Geodetic Survey is planning an expedition for supplying a series of land values in connection with the expedition, which obviously will be highly valuable as they will complete the data which have been found. Several stations on Porto-Rico, Haiti, Cuba and on the smaller islands will be occupied.

We mentioned already that South of Porto-Rico anomalies of about +40 m.d. have been found. Resuming the results near the Nares deep, we see that apparently the great negative anomalies above the deep are not accompanied by considerable positive anomalies in the neighbourhood, unless the observations on the island of Porto-Rico will reveal any such anomalies. In this regard the gravity field over this deep shows a different character from the fields which have been found during the voyage of the K XIII over several West-Pacific deeps, the Nero deep near Guam, the Yap deep and the Philippine deep. They all show a strong negative anomaly above the deep, but strong positive anomalies besides the deep; generally the anomaly to the West side of the deep, i.e. the island side, is greater than the anomaly on the Eastern side, the ocean side. Those positive anomalies diminish only slightly at a greater distance of the deep; they do not disappear.

The Java deep South of the island of Java, shows again another gravity field. This deep consists of two parallel deeps separated by a submarine ridge; the Northern one is much less deep than the Southern one. The gravity anomaly field is strongly positive on the Southern edge of the island, shows a negative minimum below the submarine ridge and gets positive again on the Southern edge of the Southern deep; again the positive anomaly on the island side is greater than on the ocean side.

Reserving a further study of these gravity fields for another occasion,

we will only remark here that they show characteristic differences, so that these deeps apparently cannot be looked at as features in identical circumstances.

Another result of the last expedition, which is worth mentioning in connection with the Nares deep, is that two stations, North of the island of Haiti, both situated to the West of the deep and in a line with it, show great negative anomalies ($\pm$ —120 m.d.) and even further to the West, in two stations North of East Cuba, the anomalies are still negative: $\pm$ —40 m.d. This seems to indicate that the stresses in the earth's crust, which are related to the formation of the Nares deep, are not confined to the region of the deep itself, but continue much farther Westward. The topography does not reveal this.

The anomalies found above and near the Eastern part of the Bartlett deep, South of East Cuba, are smaller than those of the Nares deep, which have been mentioned. Above the deep itself have been found anomalies of —61 m.d. and +13 m.d., to the North of the deep, near the coast of Cuba +24 m.d. and +56 m.d. and to the South of the deep —14 m.d., +24 m.d. and +24 m.d. Apparently the disturbances of equilibrium are smaller than above the Nares deep.

Lastly the results may be given in this connection for the sea between St. Thomas and St. Croix. The sea-floor shows here a narrow depression with a depth of about 2400 fathoms (4500 m.), while the distance between the ridges of St. Thomas and St. Croix is not more than 40 miles. Gravity in St. Thomas harbour, North of the depression, gave no anomaly and on the ridge South of it, to the West of St. Croix, an anomaly of —11 m.d.; between those stations a negative anomaly has been found of —61 m.d.

It will be necessary carefully to investigate the isostatic reductions, which have been applied to these values, in order to see if another distribution of the compensating masses may perhaps account for these results. If not, we have here a clear instance of a departure of equilibrium, which is related to a topographic feature and which probably may be interpreted as the effect of a folding or faulting process, which is going on and which presses the bottom of the depression in a position below its equilibrium position.

The gravity found in the Atlantic Ocean on the way back from Guantanamo to Washington between Crooked Island passage and Cape Hatteras gives for the three stations above deep water slightly positive anomalies; their mean value is +10 millidynes. These stations are all near the bottom of the continental slope. In Crooked Island passage, at the top of the slope, an anomaly of —26 m.d. has been found. The two stations of the U. S. Coast and Geodetic Survey which are near the top of the continental slope for this part of the coast, Beaufort, N. C. and Wilmington, N. C. likewise show negative anomalies, —21 m.d. and —31 m.d. We find here an analogous result, although less pronounced, as has been found in 1926 with the K XIII on the West Coast of Central and North America between Panama and San Francisco; the value of the anomaly (with


---- ROUTE OF THE U.S.S. S-21
.... ROUTE OF HR. MS. K XIII (1926)
GULF OF MEXICO
SIGSBEE DEEP
CAMPECHE BANK
GULF OF CAMPECHE
Bay of Campeche
YUCATAN PENINSULA
BRITISH HONDURAS
GUATEMALA
HONDURAS
SALVADOR
NICARAGUA
COSTA
Variation increasing 1' annually
Variation increasing 2' annually
New Orleans
Galveston Bay
Corpus Christi
Tampa
Charleston
Jacksonville
Miami
Tampico
Vera Cruz
Golfo de Tehuantepec
Golfo de Batabano
Straits of Florida
Bay Islands
Gulf of Honduras

Baltimore
WASHINGTON
Delaware Bay
Chesapeake Bay
Norfolk
Albemarle Sd
Pamlico Sound
Hatteras
Beaufort
C Lookout
C Fear
Variation increasing 5' annually
Bermuda
Variation increasing 6' annually
Variation increasing 4' annually
B A H A M A I S L A N D S
Great Abaco I
Eleuthera I
Andros I
Exuma Sound
Great Exuma
San Salvador
Samana Cay
Great Inagua I
Caicos Is
Grand Caicos
Turks Islands
MARIGUANA PASSAGE
CAICOS PASSAGE
TURKS PASSAGE
MOUCHOIR PASSAGE
SILVER BANK PASSAGE
WINDWARD PASSAGE
MONA PASSAGE
ISLAND OF HAITI
DOMINICAN REPUBLIC
HAITI
Port au Prince
Santo Domingo
PORTO RICO
VIRGIN ISLANDS
ANEGADA PASSAGE
JAMAICA
Kingston
Variation increasing 6' annually
NO VARIATION
C A R I B B E A N S E A
Guadeloupe
Dominica
Martinique
St Lucia
St Vincent
Grenada
Barbuda
Antigua
Golfo de Venezuela
Caracas
Variation nearly stationary

regard to sign) is greater above the foot of the slope than above the top. In the last case gravity above the top was about normal while the mean anomaly above the foot of the slope was +65 m.d.

An interpretation of this result would be premature but two remarks can be made. Firstly, it appears difficult to account for it by assuming another location of the compensating masses in the earth's crust than that which has been used for the isostatic reduction ; when examining the question the writer did not succeed in finding any acceptable mass distribution which would explain it. It looks as if we are forced to admit a deviation of equilibrium on these coasts.

Secondly, an explanation of the deviation of equilibrium on the West Coast by a Westward drift of the American continent and consequently the exertion of pressure on the ocean floor, does not seem to be in harmony with the results on the East coast, which are brought back by the last expedition. The reverse ought then to have been found on the East coast, viz. a negative anomaly above the foot of the slope, corresponding to a sucking effect behind the moving continent.

Before and after the expedition base observations have been made with the apparatus as well in the gravity base station, Washington, of the U. S. Coast and Geodetic Survey, as in the Netherlands gravity base station, De Bilt. These observations provide therefore a new check on the comparison of Washington with the international base station Potsdam. The final computations and the application of the final corrections of the time-signals have to be awaited before any conclusions will be possible.

The expedition of the U. S. S. S-21 has been made possible by the cooperation of the U. S. Navy with the Carnegie Institution and the Netherlands Geodetic Commission. Sincere thanks may be expressed to the Secretary of the U. S. Navy, the Hon. CURTIS D. WILBUR, to Admiral HUGHES, Chief of Naval Operations, to Admiral LEIGH, Chief of the Bureau of Navigation and to Captain FREEMAN, Superintendent of the Naval Observatory to whom the writer is feeling much indebted for what he has done in preparing the expedition. The writer wishes to take this opportunity for acknowledging the kind reception which has been given to hem everywhere, in Washington in naval as well as in scientific circles, on board the U. S. S. S-21 and the other ships, and ashore in the different ports which have been touched.

He expresses the hope that the expedition may lead to further research in this direction by the U. S. Navy ; results of great extent and great importance for geodetic and geophysical science might then be expected.

Mathematics. — *Zwei spezielle Kongruenzen von kubischen Raumkurven.* Von Prof. JAN DE VRIES.

(Communicated at the meeting of February 23, 1929).

1. Die kubischen Raumkurven ϱ^3 durch vier Punkte B_k, welche den Kegelschnitt α^2 zweimal treffen, bilden eine Kongruenz Γ[1]).

Es sei B_5 ein beliebig gewählter Punkt. Die kubischen Raumkurven durch die fünf Punkte B bilden eine Kongruenz von REYE; diese wird erzeugt durch zwei Büschel von quadratischen Kegelflächen deren Basis aus den Geraden B_1B_2, B_1B_3, B_1B_4, B_1B_5 bez. aus den Geraden B_2B_1, B_2B_3, B_2B_4, B_2B_5 besteht. Diese Büschel bestimmen auf α^2 je eine Involution I^4. Von den 9 gemeinschaftlichen Paaren dieser I^4 werden 3 durch Ebenenpaare erzeugt; das Paar $(B_1B_2B_3, B_1B_4B_5)$ bestimmt mit dem Paare $(B_2B_1B_3, B_2B_4B_5)$ die Figuren, welche aus der Geraden B_4B_5 und einem Kegelschnitte eines in $B_1B_2B_3$ liegenden Büschels bestehen. In diesem Büschel gibt es aber keine Kurve, welche α^2 zweimal trifft.

Demnach liegt ein beliebig gewählter Punkt des Raumes auf *sechs* Kurven ϱ^3 der Kongruenz Γ.

2. *a*. Durch zwei Punkte A', A'' von α^2 und die vier Punkte B_k $(k=1, 2, 3, 4)$ geht eine ϱ^3; als ihr Bild betrachte ich den Schnittpunkt E der Geraden a', a'' welche α^2 in A', A'' berühren.

In der Ebene β_4 durch B_1, B_2, B_3 liegt ein Kegelschnitt β_4^2 durch diese drei Punkte, welcher α^2 in A'_4, A''_4 trifft; er bildet mit jeder durch B_4 gelegten, ihn treffenden Geraden b_4 eine Figur von Γ. Alle diese Figuren werden abgebildet in den *singulären Punkt* S_4, den Schnittpunkt der Tangenten a'_4, a''_4. Analog gibt es die *singulären Punkte* S_1, S_2, S_3.

b. Die Gerade a'_4 enthält die Bilder der Figuren, welche zusammengesetzt sind aus einem Kegelschnitt γ_4^2 durch A'_4, B_1, B_2, B_3 und einer Geraden c_4 durch B_4, welche γ_4^2 und α^2 trifft. Den *acht Tangenten* a'_k, a''_k entsprechen *acht Systeme* von zusammengesetzten Figuren. Jeder Kurve γ_4^2 werden *drei* Geraden c_4 zugeordnet.

c. Jeder Kegelsschnitt ϱ^2 durch B_1 und B_2, welcher α^2 zweimal trifft und b_{34} (B_3B_4) schneidet, bildet mit b_{34} eine Figur von Γ. Der Ort der ϱ^2 ist eine kubische Fläche F_{12}^3, denn die Gerade b_{12} bildet mit einer Sehne von α^2 eine Figur ϱ^2. Weil die Ebenen durch b_{12} auf α^2 eine I^2

[1]) Herr Dr. VAN KOL hat in seiner Dissertation u. A. die Kongruenz $[\varrho^3]$ untersucht, wo α^2 durch eine rationale Raumkurve α^n ersetzt ist. Es dürfte sich aber empfehlen den einfachen Fall $n=2$ gesondert zu betrachten.

bestimmen, wird das System dieser zusammengesetzten ϱ^3 abgebildet auf eine Gerade d_{12}. Diese Gerade enthält die Punkte S_1, S_2, denn die Ebenen B_3b_{12} und B_4b_{12} enthalten je einen Kegelschnitt β_k^2.

Die Systeme c werden also abgebildet auf die Seiten d des durch die Punkte S bestimmten vollständigen Vierecks, durch dessen Eckpunkte je zwei Bildgeraden a', a'' der Systeme b gehen.

Der Punkt $d_{12}d_{34}$ ist das Bild einer aus drei Geraden zusammengesetzten ϱ^3, nämlich aus b_{12}, b_{34} und der sich auf ihnen stützenden Sehne der α^2. Die 24 Schnittpunkte da' und da'' sind ebenfalls Bilder von dreiteiligen Figuren; auch die 24 Schnittpunkte von zwei Geraden a. Im Ganzen enthält Γ daher 51 *drieteilige Figuren.*

3. Die Kongruenzkurven, welche α^2 in einem Punkte A treffen, bilden eine Fläche F^7. Die ϱ^3 durch A und B_k, welche eine Gerade treffen, bilden bekanntlich [1]) eine Fläche F^5, mit 3-fachen Punkten A und B_k; es gibt demnach 7 Kurven ϱ^3, welche α^2 noch in einem zweiten Punkt treffen.

Das System Λ der ϱ^3, welche l treffen, wird somit abgebildet auf eine Kurve λ^7. Diese hat Doppelpunkte in S_k, denn l trifft zwei Geraden b_k (§ 2). Zwei Kurven λ^7 haben somit 33 Bildpunkte E gemein.

Der *Ort* der ϱ^3 des Systems Λ ist daher eine Fläche Λ^{33}, mit *sechsfacher* Geraden l (§ 1) und *siebenfacher* Kurve α^2.

Der Schnitt von Λ^{33} mit β_4 besteht aus den Doppelkurve β_4^2, zwei dreifachen Kurven γ_4^2, vier einfachen γ_4^2 und den dreifachen Geraden b_{12}, b_{13}, b_{23}. Hieraus erhellt, dass B_k 18-facher Punkt ist.

4. Jede Kongruenzkurve δ^3, welche α^2 *dreimal* trifft, liegt auf dem durch α^2 und die Punkte B gelegten Hyperboloid H^2. Es gibt demnach zwei Büschel (δ^3), deren jede die Geraden einer Regelschaar zu Bisekanten hat.

Die beide Systeme werden abgebildet auf zwei Kegelschnitte δ^2, welche je Involutionskurven von Tangenteninvolutionen der α^2 sind, und sich in den singulären Punkten S durchsetzen.

Die Fläche F^7 enthalt daher offenbar zwei Doppelkurven ϱ^3.

Die 6 Punkte E in denen δ^2 durch λ^7 getroffen wird, sind die Bilder der beiden δ^3, welche l schneiden. Die Fläche Λ^{33} enthält somit *vier dreifache Kurven* δ^3; diese entsprechen den Schnitten von l mit H^2.

5. Der Kegelschnitt φ^2, den eine Ebene φ mit H^2 gemein hat, enthält zwei I^3, welche durch die beiden Büschel (δ^3) erzeugt werden. In jedem Büschel gibt es somit vier ϱ^3, welche φ berühren; die Bildkurve des Systems Φ der von φ berührten Kurven hat demnach mit δ^2 vier Tripel von Bildpunkten E gemein. Da sie in S_k *vierfache* Punkte hat (diese

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[1]) Der Schnitt mit einer Ebene β besteht aus einem Kegelschnitt und drei Geraden.

entsprechen den Schnitten von φ mit β_k^2) beläuft die Anzahl ihrer Schnittpunkte mit δ^2 28. Demnach wird Φ abgebildet auf eine Kurve φ^{14} (S_k^4).

Mit λ^7 (S_k^2) hat sie 66 Punkte E gemein; demnach ist der *Ort* der von φ berührten ϱ^3 eine Fläche F^{66}, mit *vierfachen* Kurven β_k^2, acht *dreifachen* Kurven ϱ^3 und der *vierzehnfachen* Kurve α^2. (Eine Tangente a enthält 14 Punkte E der φ^{14}).

Zwei Ebenen werden von 132 Kongruenzkurven berührt.

6. Die quadratischen Flächen durch die vier Punkte B und die Gerade l bilden ein Netz; sie erzeugen auf α^2 eine Involution I_2^4. Jedes der drei neutralen Paare entspricht den Flächen eines Büschels (F^2), dessen Basis aus l und einer ϱ^3 von Γ besteht.

Somit ist eine beliebig gewählte Gerade Bisekante von *drei* Kongruenzkurven und Λ^{33} hat die *dreifache* Gerade l.

7. Es soll noch die Kongruenz $\triangle$ untersucht werden, welche die Kurven ϱ^3 durch B_1, B_2, B_3 enthält, die den in der Ebene a liegenden Kegelschnitt α^2 und die Gerade c je zweimal treffen.

Die Kurven ϱ^3 durch B_k und einen Punkt A von α^2 welche c zur Bisekante haben und l schneiden, bilden eine Fläche F^4, denn die Ebene β (durch B_k) enthält zwei Kegelschnitte als Bestandteile von kubischen Kurven. Da A offenbar Doppelpunkt von F^4 ist, trifft α^2 diese Fläche noch in 6 Punkten A'; durch A gehen somit *sechs* Kurven von $\triangle$.

Wird eine Kurve von $\triangle$ wiederum abgebildet in den Schnittpunkt E der Geraden a', a'', welche α^2 in Punkten A', A'' berühren, in denen α^2 von jener ϱ^3 getroffen wird, so bildet eine Tangente a die Kurven ϱ^3 ab, welche α^2 in einem Punkte A treffen; diese Kurven bilden daher eine Fläche F^6.

Das System der ϱ^3, welche eine Gerade l schneiden, wird folglich auf eine Kurve λ^6 abgebildet:

8. *a*. In der Ebene β liegt ein Büschel von nodalen kubischen Kurven k^3, welche ihren Doppelpunkt in der Spur C^* von c haben, durch B_k gehen und α^2 in A_0', A_0'' treffen. Dieses zu $\triangle$ gehörende System wird in den *singulären Punkt* S_0 $(a_0' a_0'')$ abgebildet.

b. In der Ebene γ_1 $(B_1 c)$ liegt ein Büschel γ_1^2, dessen Basis aus B_1, der Spur B_{23} von b_{23} $(B_2 B_3)$ und den Spuren A_1', A_1'' von α^2 besteht. Jede Kurve γ_1^2 wird durch b_{23} zu einer ϱ^3 ergänzt. Dieses System wird in dem *singulären Punkt* S_1 abgebildet.

Analog gibt es die singulären Punkte S_2 und S_3. Die *singulären Punkte* S_1, S_2, S_3 liegen offenbar auf der *Polare* c_0 von C_0 (ca) in Bezug auf α^2.

c. Die Ebene β enthält einen Kegelschnitt β'^2 durch B_1, B_2, B_3, A_0' und die Spur C^* von c. Diese Kurve wird zu Kongruenzkurven ergänzt durch jede Transversale r' von β'^2, c und α^2. Dieses System von Figuren ϱ^3 wird abgebildet auf die Tangente a_0'.

Die Geraden r' bilden eine Regelfläche $(r')^5$, auf welcher β'^2 *Doppelkurve, c dreifache* Gerade ist.

Weil die Figur $(\beta'^2, C A_0'')$ dem Büschel (k^3) angehört, wird sie in S_0 abgebildet.

Analog bildet die Punktreihe auf a_0'' das System ab, welches aus dem Kegelschnitt β''^2 und Transversalen r'' von β''^2, c und α^2 besteht.

d. In der Ebene β liegt ferner ein Büschel (β^2), durch B_k und C^*, von welchem jede β^2 zu einer ϱ^3 ergänzt wird durch jede der beiden Sehnen r_1, r_2 van α^2, welche β^2 treffen. Weil der Stralenbüschel um C_0 (dem r_1, r_2 angehören) auf α^2 eine I^2 bestimmt, wird dieses System von $\triangle$ auf eine Gerade abgebildet, und zwar auf die Gerade c_0; denn es enthält eine Figur, welche aus b_{kl} und einem Geradenpaar der Ebene γ_m $(B_m\, c)$ besteht.

e. Jeder Kegelschnitt ϱ_{12}^2 durch B_1 und B_2, welcher c schneidet und α^2 zweimal trifft, bildet eine Figur ϱ^3 mit der Geraden t_3 durch B_3, welche c und ϱ_{12}^2 trifft. Jede Gerade t_3 gehört zwei Figuren ϱ^3 an, denn die Kurven ϱ_{12}^2 bilden eine Fläche F_{12}^3, welche c (und b_{12}) enthält.

Zu diesem System gehört eine ϱ^3, welche aus dem Kegelschnitt $B_1B_2A_0'A_0''C^*$ und der Geraden B_3C^* besteht, also in S_0 abgebildet wird. Die Bildgerade d_{12} des Systems geht demnach durch S_0.

f. Durch B_3 gehen zwei Transversalen, t_3' und t_3'', über c und α^2. Jeder Kegelschnitt $\tau_{12}'^2$ durch B_1, B_2, welcher t_3', c und α^2 trifft, bildet mit t_3' eine ϱ^3.

Der Ort dieser Kegelschnitte ist eine Fläche $T_{12}'^5$; denn b_{12}, c, t_3' und α^2 haben eine Transversale, welche mit b_{12} ein Geradenpaar bildet, und jede Ebene durch b_{12} enthält zwei $\tau_{12}'^2$; B_1 und B_2 sind dreifache Punkte.

In diesem System gibt es eine dreiteilige ϱ^3, welche aus t'_3, b_{12} und deren durch A_3'' gelegten Transversale besteht, also in S_3 abgebildet wird. Demnach wird das System auf die Punktreihe von a'_3 abgebildet.

Analog ist a_3'' die Bildgerade des Systems, welches der Geraden t_3'' entspricht.

Vermittelst der Bilder der zusammengesetzten Kurven, lässt sich unschwer die Anzahl der dreiteiligen Figuren bestimmen. Die Punkte S_k bilden je drei von ihnen ab; es gibt deren 18 mit Bildpunkten da, 24 mit Bildern $a_k a_l$ und schliesslich 3 mit Bildern $c_0 d$. Im Ganzen gibt es somit 48 *dreiteilige Figuren* in $\triangle$.

9. Es sei δ^3 eine Kurve von $\triangle$, welche α^2 *dreimal* trifft. Das Netz der quadratischen Flächen durch A_1, B_k und c erzeugt auf α^2 eine Involution I_2^3; wenn A_2, A_3 das neutrale Paar bilden, so gibt es in jenem Netze einen Büschel, dessen Basis aus c und einer Kurve δ^3 besteht. Die Kurven δ^3 treffen α^2 daher in den Tripeln einer I^3; folglich werden sie abgebildet auf die Tripel eines Kegelschnitts δ^2.

Weil δ^2 mit λ^6 12 Bildpunkte gemein hat, welche offenbar 4 Tripel bilden, ist der *Ort* der Kurven δ^3 eine Fläche $\triangle^4$. Ihr Schnitt mit a

besteht aus α^2 und den Geraden $C_0 A_0'$, $C_0 A_0''$; die diesen Geraden entsprechenden Kurven β^2 bilden den Schnitt von $\triangle^4$ mit β. Die Punkte B sind *Doppelpunkte*, c ist *Doppelgerade* von $\triangle^4$; ihr Schnitt mit der Ebene $B_k c$ besteht aus c und zwei Geraden.

10. Weil die Bildkurve λ^6 die vier singulären Punkte S enthält, gibt es 32 ϱ^3, welche zwei beliebig gewählte Geraden schneiden, und der *Ort* der l treffenden ϱ^3 ist eine Fläche Λ^{32}, mit *sechsfacher* Kurve α^2 und *fünfzehnfacher* Geraden c. Letzteres erhellt aus § 3; denn die ϱ^3 durch B_k und einen Punkt C von c, welche α^2 zweimal treffen und sich auf eine Gerade stützen, bilden eine Fläche F^{33}, mit 18-fachem Punkt C, wonach es 15 Kurven von $\triangle$ gibt, die c noch ausserhalb C treffen.

Der Schnitt von Λ^{32} mit β besteht aus 2 fünffachen ϱ^2, einem doppelten und einem einfachen Kegelschnitt, 3 Geraden und einer nodalen k^3. Demnach sind die Punkte B 16-fache Punkte der Λ^{32}.

11. Der quadratische Kegel durch c, welcher B_1, B_2, B_3 und A_1 aus einem Punkte C projiziert, trifft α^2 noch in 3 Punkten A_2, enthält somit 3 Kurven von $\triangle$. Die ϱ^3, welche c in C treffen, werden daher abgebildet auf eine Kurve $\gamma^3 (S_1\, S_2\, S_3)$.

Sie hat mit δ^2 zwei Tripel, mit λ^6 15 Punkte E gemein; hieraus erhellt wieder, dass c Doppelgerade von $\triangle^4$ und 15-fache Gerade von Λ^{32} ist.

Die Kongruenzkurven durch C bilden eine Fläche F^{15}, mit *dreifacher* Kurve α^2 und 2 *dreifachen* Kurven ϱ^3. Ihr Schnitt mit β besteht aus den Geraden b_{12}, b_{13}, b_{23} und den dreifachen Kegelschnitten β'^2, β''^2 (welche den Schnitten der γ^3 mit a_0', a_0'' entsprechen). Demnach sind B_k *achtfache* Punkte von F^{15}.

12. Auf der Fläche F^6, welche der Ort ist der ϱ^3 durch den Punkt A von α^2, ist c ersichtlich dreifache Gerade und die Kurve δ^3 durch A eine Doppelkurve.

Die Schnittkurve φ^6 mit einer Ebene φ hat somit einen dreifachen Punkt und 3 Doppelpunkte, daher das Geschlecht 4. Die ϱ^3 von F^6 erzeugen auf φ^6 eine I^3, welche folglich 12 Doppelpunkte besitzt. Die ϱ^3, welche φ berühren, werden also abgebildet auf eine Kurve φ^{12}.

Diese $\varphi^{12} (S_0^4, 3\, S_k^2)$ hat mit $\lambda^6 (S_0, 3\, S_k)$ 62 Punkte E gemein. Demnach bilden die ϱ^3, welche φ berühren, eine *Fläche* Φ^{62}, mit 12-facher α^2 und 30-facher c (aus φ^{12} und γ^3).

Der Schnitt dieser Fläche mit β setzt sich zusammen aus 4 nodalen Kurven k^3, den 8-fachen Kurven β'^2, β''^2, drei Doppelkurven β^2 und den Doppelgeraden b_{kl}; im Ganzen ein Schnitt vom Grade 62. Hieraus ergibt sich wieder dass c 30-fache Gerade ist, und dann dass B_k 32-fache Punkte sind.

Zwei Ebenen werden von 116 Kongruenzkurven berührt.

Mathematics. — *Ueber die in der Wellengleichung verwendeten hyperkomplexen Zahlen.* Von J. A. SCHOUTEN. (Communicated by Prof. P. EHRENFEST.)

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In seiner Arbeit „A symmetrical treatment of the wave equation" [1]) hat A. S. EDDINGTON gezeigt, dasz die bekannten Asymmetrien in den Gleichungen von DIRAC entfernt werden können wenn man mit der Theorie der Matrizen anfängt. Obwohl diese Methode, die inzwischen zur mathematischen Festlegung der Naturkonstante $hc/2\pi e^2$ geführt hat [2]), wohl als eine bedeutende Verbesserung anerkannt werden musz, entbehrt sie noch der sichern Grundlage, da sie sich auf gewisse merkwürdige zu Anfang eingeführte uneigentliche "Drehungen" aufbaut, die nur dazu dienen, den Operatoren $E_1, \ldots, E_5$ die ihnen zukommenden Eigenschaften zu verschaffen, und dementsprechend später gar nicht mehr verwendet werden.

Es soll nun im Folgenden gezeigt werden, dasz die uneigentlichen Drehungen als Grundlage vollständig entbehrt werden können, sobald man noch einen Schritt weiter zurück geht und mit der Theorie der komplexen Zahlensysteme anfängt. Es zeigt sich dabei, dasz das kleinste Zahlensystem, das überhaupt in Frage kommen kann, das "ursprüngliche" System mit 16 Einheiten ist und dabei ergeben sich die Eigenschaften der Operatoren E, einschlieszlich der Möglichkeit der Matrixdarstellung, aus den bekannten Eigenschaften dieser Systeme.

1. Es seien E_1, E_2, E_3, E_4 vier höhere komplexe Zahlen mit den Rechenregeln

$$\left.\begin{aligned} E_i\,E_i &= 1 \\ E_i\,E_j &= -E_j\,E_i \end{aligned}\right\} i, j = 1, \ldots, 4\,, \qquad (1)$$

die auszerdem dem *assoziativen* Gesetz unterworfen sind. Dann folgt, dasz die 16 Zahlen $1, E_1, \ldots, E_{12} = E_1 E_2, \ldots, E_{123} = E_1 E_2 E_3, \ldots, E_{1234} = E_1 E_2 E_3 E_4$ ein geschlossenes assoziatives System bilden. Es ist das vierte System in der Reihe der sogenannten ursprünglichen Systeme [3]), d.s. Systeme die kein invariantes Untersystem enthalten, und wir wollen dasselbe dementsprechend mit U_4 bezeichnen. Die aus (1) hervorgehende reelle Form der Rechenregeln (d.h. Form mit reellen Koeffizienten) führt auf CLIFFORD zurück.

[1]) Proc. Roy. Soc. London, A 121, (28), 524—542.

[2]) The charge of an electron, Proc. Roy. Soc. London, A. 122, (29), 358—369.

[3]) Auch System der Sedenionen oder Quadriquaternionen genannt.

Eine zweite reelle Form, die sich aber aus der ersten vermöge komplexer Transformationen ableitet, wird gebildet durch die Produktregeln der 4-reihigen Matrizen (genauer: der gemischten Gröszen zweiten Grades in vier dimensionen)

$$\sum_j P_i^{\cdot j} Q_j^{\cdot k} = R_i^{\cdot k} \quad ; \quad i, j, k = 1, \ldots, 4 \quad \ldots \ldots \quad (2)$$

Aus dem assoziativen Gesetz folgt also, dasz sich die 16 Zahlen E als 4-reihige Matrizen auffassen lassen.

Eine dritte, mit *der zweiten* ebenfalls in komplexer Weise, aber mit *der ersten* in reeller Weise verknüpfte reelle Form entsteht, indem man bemerkt, dasz E_{1234} sich ebenfalls antikommutativ verhält zu E_1, E_2, E_3 und E_4 während $E_{1234}\,E_{1234} = 1$ ist. Schreibt man nun $E_{1234} = E_5$, so ist

$$\left.\begin{array}{l} E_i\,E_i = 1 \\ E_i\,E_j = -E_j\,E_i \end{array}\right\} i, j = 1, \ldots, 5, \quad \ldots \ldots \quad (3)$$

und es ergeben sich daraus die Rechenregeln für die 16 Einheiten $1, E_1, \ldots, E_{12} = E_1\,E_2, \ldots$

Diese Regeln führen ebenfalls auf CLIFFORD zurück. Die merkwürdigen Beziehungen zwischen vierdimensionaler und fünfdimensionaler Invarianz finden ihren Grund in dieser selbstergänzenden Eigenschaft der vier hyperkomplexen Zahlen E_1, E_2, E_3 und E_4. Ein System von fünf Zahlen $E_1, \ldots, E_5$, das den Regeln (3) unterworfen ist, soll ein *Orthogonalsystem* heiszen. Offenbar giebt es im System U_4 kein Orthogonalsystem das mehr als fünf Zahlen enthält.

Eine vierte reelle Form folgt aus der Eigenschaft, dasz das System U_4 das Produkt von zwei Quaternionsystemen ist. Bilden also $1, \lambda_1, \lambda_2, \lambda_3$ einerseits und $1, \mu_1, \mu_2, \mu_3$ anderseits ein Quaternionsystem mit den Rechenregeln

$$\left.\begin{array}{ll} \lambda_1\,\lambda_1 = -1 & \mu_1\,\mu_1 = -1 \\ \lambda_1\,\lambda_2 = -\lambda_2\,\lambda_1 = \lambda_3 & \mu_1\,\mu_2 = -\mu_2\,\mu_1 = \mu_3 \\ \quad\textit{cycl.} & \quad\textit{cycl.} \end{array}\right\}, \quad \ldots \quad (4)$$

so bilden $1, \lambda_i, \mu_i, \lambda_i\,\mu_j = \mu_j\,\lambda_i, \; i, j = 1, 2, 3$, ein System U_4. Es ist leicht einzusehen, dasz ein Orthogonalsystem sich etwa folgendermaszen bilden läszt

$$E_1 = \lambda_1\,\mu_3, \quad E_2 = \lambda_2\,\mu_3, \quad E_3 = \lambda_3\,\mu_3, \quad E_4 = -i\,\mu_1, \quad E_5 = -i\,\mu_2 \;. \quad (5)$$

Umgekehrt lassen sich zu jedem Orthogonalsystem zwei Systeme λ bez. μ so wählen, dasz sie den Gleichungen (5) genügen[1]):

$$\left.\begin{array}{lll} \mu_1 = i\,E_4\,, & \mu_2 = i\,E_5\,, & \mu_3 = -E_{45} \\ \lambda_1 = -E_{23}, & \lambda_2 = -E_{31}, & \lambda_3 = -E_{12} \end{array}\right\} \quad \ldots \quad (6)$$

[1]) Man bekommt die σ und ϱ von Dirac wenn man setzt: $\sigma_i = i\lambda_i$, $\varrho_i = i\mu_i$.

2. Die mit $E_1, \ldots, E_5$ korrespondierenden Matrizen sind Wurzeln der Einheitsmatrix. Sie haben infolgedessen nur lineare Elementarteiler und eine jede läszt sich mit Hilfe geeigneter Koordinatentransformationen in die Diagonalform überführen, wo in der Diagonale entweder $+1, +1, +1, -1$ oder $+1, +1, -1, -1$ stehen. Nun läszt sich aber jede der 5 Zahlen E als Differenz eines Produktes und seiner Umkehrung schreiben, z.B.

$$E_1 = {}^1/_2\,(E_1 E_{12} - E_{12} E_1), \quad \ldots \ldots \quad (7)$$

woraus nach einem bekannten Satze hervorgeht, dasz die Spur Null ist. Es bleiben also nur Matrizen mit den Elementarteillern $(\lambda-1)$, $(\lambda-1)$, $(\lambda+1)$, $(\lambda+1)$. Aus der Theorie der Elementarteiler folgt, dasz sich alle gemischten Gröszen zweiten Grades mit denselben Elementarteilern durch lineare Transformationen ineinander überführen laszen, und est ist somit sichergestellt, dasz man mit jeder beliebigen Zahl, deren Matrix die Elementarteiler $(\lambda-1)$, $(\lambda-1)$, $(\lambda+1)$, $(\lambda+1)$ besitzt, als erste Zahl eines Orthogonalsystems anfangen kann und dasz sich dann stets ein diese Zahl enthaltendes Orthogonalsystem bilden läszt.

Für die Zahlen λ und μ läszt sich ähnliches ableiten. Die Elementarteiler der zu den Zahlen aus diesen Zahlentripletten gehörigen Matrizen sind $(\lambda-i)$, $(\lambda-i)$, $(\lambda+i)$, $(\lambda+i)$ und daraus geht hervor, dasz man mit jeder beliebigen Zahl, deren Matrix diese Elementarteiler besitzt, als erste Zahl eines Zahlentriplettes anfangen kann und dasz sich dann stets ein diese Zahl enthaltendes Triplett und ein dazugehöriges zweites Triplett bilden läszt, so dasz die Rechenregeln (4) gelten.

3. Stellt man die Frage, wie viele Orthogonalsysteme existieren, die E_5 enthalten, so ist zunächst zu bemerken, dasz auch $E'_1, \ldots, E'_4, E_5$; $E'_j = iE_5 E_j$, $j = 1, \ldots, 4$, ein solches System bilden. Dieses Resultat rührt von EDDINGTON her; bei ihm bleiben aber diese beiden Systeme (bis auf Vorzeichenwechsel) die einzig möglichen, was damit zusammenhängt, dasz seine „perpendicular sets" von fünf Zahlen in Bezug auf die zu Anfang erwähnten uneigentlichen Drehungen definiert sind und sich somit nicht genau mit unseren Orthononalsystemen decken. Es berechnet sich leicht, dasz die allgemeine Form des gewünschten Systems (bis auf Vorzeichenwechsel) lautet:

$$\alpha E_1 + \beta E'_1, \ldots, \quad \alpha E_4 + \beta E'_4, E_5, \ \alpha^2 + \beta^2 = 1 \quad \ldots \quad (8)$$

Sehen wir von Vorzeichenwechsel ab, so gibt es also ∞^1 reelle Systeme, die E_5 enthalten, und diese sind einander paarweise zugeordnet (α, β zu $-\beta, \alpha$).

Jedes dieser Systeme z.B. $E_1, \ldots, E_5$ ist invariant bei orthogonalen Transformationen der fünf Operatoren. Multipliziert man eine lineare Gleichung in den E, z.B.

$$(\sum_i t_i\, E_i)\,\psi = 0 \;\; ; \;\; i = 1, \ldots, 5, \quad \ldots \ldots \quad (9$$

die in diesem Sinne fünfdimensionale Invarianz aufweist, mit E_5, so entsteht die mit (9) aequivalente

$$(\sum_i t_i E_i' + t_5)\,\psi = 0 \quad ; \quad i = 1, \ldots, 4, \quad \ldots \quad (10)$$

die vierdimensionale Invarianz besitzt. Dieses wichtige Resultat EDDINGTONS ist also unabhängig von der bei ihm zu Grunde gelegte auf die uneigentlichen Drehungen beruhenden Definition der "perpendicular sets" und ist rein eine Folge der Eigenschaften des Zahlensystems U_4.

Jede homogene quadratische Form in den $E_1, \ldots, E_5$ lässt sich als nicht homogene quadratische Form in $E_1', \ldots, E_4'$ schreiben[1]);

$$\sum_{ij} t_{ij} E_i E_j = \sum_{ab} t_{ab} E_a' E_b' + i \sum_a (t_{a5} - t_{5a}) E_a' + t_{55} \quad \ldots \quad (11)$$

$$i, j = 1, \ldots, 5\,; \; a, b = 1, \ldots, 4.$$

4. Es sei zum Schlusz die Frage erörtert welche Zahlen überhaupt die Form $\sum_i t_i E_i$, $i = 1, \ldots, 5$ annehmen können. Aus der Möglichkeit der orthogonalen Transformation der E folgt, dasz eine solche Zahl sich nur durch einen gewöhnlichen Zahlenfaktor unterscheidet von einer Zahl deren Matrix die Elementarteiler $(\lambda - 1)$, $(\lambda - 1)$, $(\lambda + 1)$, $(\lambda + 1)$ besitzt. Die gesuchten Zahlen sind also diejenigen, deren Matrizen lineare Elementarteiler und eine charakteristische Gleichung mit zwei zweimal zählenden entgegengesetzten Wurzeln besitzen.

1) Diese Bemerkung verdanke ich Herrn D. VAN DANTZIG.

Mathematics. — *Die Kongruenz der rationalen biquadratischen Raumkurven, die durch sieben gegebene Punkte gehen.* Von G. SCHAAKE. (Communicated by Prof. JAN DE VRIES.)

(Communicated at the meeting of February 23, 1929).

§ 1. In dieser Mitteilung wird die Kongruenz der rationalen biquadratischen Raumkurven k^4, die durch sieben gegebene Punkte $P_1, \ldots, P_7$ gehen, untersucht werden mit Hilfe einer Abbildung der Kurven k^4 auf die Punkte K einer Ebene α.

Um zu dieser Abbildung zu gelangen betrachten wir zuerst einen rationalen kubischen Kegel $\varkappa$ mit der Spitze T und eine Gerade d. Eine auf $\varkappa$ sich befindende rationale biquadratische Raumkurve bestimmt eine Verwandtschaft (1,4) zwischen den Ebenen δ durch d und den Erzeugenden b von $\varkappa$, wenn wir eine Ebene δ und eine Erzeugende b, die sich in einem von T verschiedenen Punkte dieser Kurve begegnen, immer einander zuordnen. Drei der Geraden b, die der Ebene dT korrespondieren, fallen zusammen mit den Geraden TD_1, TD_2 und TD_3, wenn D_1, D_2 und D_3 die Schnittpunkte von d mit $\varkappa$ sind.

Eine allgemeine einvierdeutige Verwandtschaft zwischen den Ebenen δ und den Geraden b bestimmt umgekehrt eine Kurve siebenter Ordnung k^7 auf $\varkappa$, die durch D_1, D_2 und D_3 geht und in T einen vierfachen Punkt hat. Wenn in dieser Verwandtschaft der Ebene dT die drei Geraden TD_1, TD_2 und TD_3 entsprechen, so lösen diese sich von k^7 ab und es bleibt eine zu $\varkappa$ gehörende rationale Kurve k^4 übrig.

Hieraus folgt, dass eine rationale Kurve k^4 auf $\varkappa$ durch $4+4+1-3=6$ Punkte von $\varkappa$ bestimmt ist.

Wir projizieren eine rationale Kurve k^4 durch $P_1, \ldots, P_7$ mittelst des kubischen Kegels $\varkappa$ aus P_7 auf α. Die Projektion ist eine nodale kubische Kurve k'^3, die durch die Projektionen $P'_1, \ldots, P'_6$ von $P_1, \ldots, P_6$ geht. Wir bilden k^4 ab auf den Doppelpunkt K von k'^3.

Ein willkürlicher Punkt K von α ist dann der Bildpunkt einer durch $P_1, \ldots, P_7$ gehenden Kurve k^4. Denn es gibt eine Kurve k'^3 durch $P'_1, \ldots, P'_6$, die in K einen Doppelpunkt hat und der Kegel $\varkappa$, der k'^3 aus P_7 projiziert, enthält eine rationale Kurve k^4, die durch $P_1, \ldots, P_6$ geht; diese Kurve geht auch durch die Spitze P_7 von $\varkappa$.

Der Bildpunkt K einer Kurve k^4 durch $P_1, \ldots, P_7$ ist der Spurpunkt der durch P_7 gehenden Trisekante von k^4. Diese Trisekante liegt auf der durch k^4 bestimmten Fläche φ^2.

Man findet also die einem gegebenen Punkte K von α entsprechende

Kurve k^4 auch mit Hilfe der quadratischen Fläche φ^2, die die Punkte $P_1, \dots, P_6$ und die Gerade P_7K enthält, wenn man hierauf die rationale biquadratische Kurve durch $P_1, \dots, P_7$ konstruiert, die P_7K dreimal schneidet.

Die Kurve k^4 ist auch der von P_7K verschiedene Durchschnitt des Kegels $\varkappa$ und der Flache φ^2, die aus K abgeleitet sind.

§ 2. Wir denken uns die durch $P_1, \dots, P_6$ bestimmte kubische Raumkurve k^3. Die durch P_7 gehende Bisekante b von k^3 schneidet α in S'. Dieser Punkt ist der Bildpunkt aller Kurven k^4, welche aus k^3 und einer k^3 schneidenden Gerade s durch P_7 bestehen.

Der Punkt S' ist ein singulärer Punkt für unsere Abbildung. Die auf S abgebildeten Kurven k^3 bestehen aus k^3 und den Erzeugenden s des kubischen Kegels $\varkappa_7$, der k^3 aus P_7 projiziert.

Es gibt ∞^1 Kurven k'^3, die in P'_1 einen Doppelpunkt haben. Diese Kurven sind die Projektionen der Kurven k^4, die $P_7 P_1$ dreimal schneiden und alle auf P'_1 abgebildet werden. Wenn wir von einer bestimmten dieser Kurven k'^3 ausgehen, dann ist die zugeordnete Kurve k^4 der von $P_1 P_7$ verschiedene Durchschnitt des projizierenden Kegels $\varkappa$ der Kurve k'^3 und der quadratische Fläche φ^2, die $\varkappa$ in P_1 berührt. Die auf P'_1 abgebildeten Kurven k^4 sind also Durchschnitte von zwei sich entsprechenden Flächen in einer Verwantschaft (1, 2) zwischen dem Büschel der Kegel $\varkappa$, von denen $P_1 P_7$ die Doppelgerade ist und dem Büschel von Flächen φ^2, deren Basiskurve gebildet wird durch die Gerade $P_1 P_7$ und die Kurve k_{17}^3, die durch $P_2, \dots, P_6$ geht und $P_1 P_7$ zweimal schneidet. Die betrachteten Kurven k^4 bilden also eine Fläche siebenter Ordnung φ_1^7 welche die Geraden $P_2 P_7, \dots, P_6 P_7$ enthält, wovon die Kurve k_{17}^3 Doppelkurve und $P_1 P_7$ vierfache Gerade ist und wofür P_7 und P_1 fünffache und die anderen Punkte P dreifache Punkte sind.

Die Gerade $P_1 P_7$ ist doppelte torsale Erzeugende von φ_1^7, weil die aus $P_1 P_7$ und k_{17}^3 bestehende Kurve Doppelkurve ist unserer Kongruenz. Von jedem Punkte von $P_1 P_7$ gehen also noch zwei Kurven k^4 von φ_1^7 aus. Die einzweideutige Verwandtschaft zwischen den Büscheln von Flächen φ^2 und Kegeln $\varkappa$, die jedesmal die Flächen φ^2 und $\varkappa$ einander zuordnet, welche sich in einem festen Punkte von $P_1 P_7$ berühren, hat mit der betrachteten Verwandtschaft (1, 2) dann auch vier Paare gemein, wovon zwei gebildet werden durch die beiden quadratischen Kegel, die k_{17}^3 aus ihren beiden Schnittpunkten mit $P_1 P_7$ projizieren und dem Kegel $\varkappa$, der k_{17}^3 aus P projiziert.

Unsere Abbildung besitzt also noch sechs singuläre Punkte $P'_1, \dots P'_6$. Die auf P'_i abgebildeten Kurven k^4 bilden eine Flache φ_i^7, welche die Verbindungsgeraden von P_7 mit den von P_i und P_7 verschiedenen Punkten P enthält, wovon $P_i P_7$ Doppelgerade und k_{i7}^3 Doppelkurve ist und wofür P_7 und P_i fünffache und die anderen Punkte P dreifache Punkte sind.

§ 3. Es gibt sechs Systeme $\Sigma_1, \ldots, \Sigma_6$ von ausgearteten Kurven k^4, die analog sind dem Systeme Σ_7, das auf S' abgebildet wird. Das System Σ_1, zum Beispiel, besteht aus der Kurve k_1^3 durch $P_2, \ldots, P_7$ und den Erzeugenden des Kegels $\varkappa_1$, der k_1^3 aus P_1 projiziert. Seine Kurven werden abgebildet auf die Punkte des Kegelschnittes $k_1'^2$, der durch $P_2', \ldots, P_6'$ bestimmt ist.

Weiter gibt es 21 Systeme Σ_{ik} von ausgearteten Kurven k^4. So bestehen alle Kurven des Systems Σ_{12} aus der Geraden $P_1 P_2$ und den kubischen Raumkurven durch $P_3 \ldots, P_7$, die $P_1 P_2$ schneiden. Die 15 Systeme Σ_{ik}, wofür keine der Zahlen i und k gleich 7 ist, werden abgebildet auf die Geraden $P_i' P_k'$. Jede dieser Geraden enthält zwei Punkte, die Bildpunkte sind der dem entsprechenden Systeme Σ_{ik} zugehörenden Doppelkurve unserer Kongruenz. Diese besteht aus $P_i P_k$ und der Kurve k_{ik}^3 durch die von P_i und P_k verschiedenen Punkte P, die $P_i P_k$ zweimal schneidet. Der Ort der kubischen Bestandteile der Kurven von Σ_{ik} ist eine Flache fünfter Ordnung Ω_{12}^5, die dreifache Punkte hat in den von P_i und P_k verschiedenen Punkten P, wofür k_{ik}^3 Doppelkurve ist und die $P_i P_k$ enthält.

Die sechs Systeme Σ_{ik}, wofür eine der Zahlen i und k gleich 7 ist, werden jedesmal auf eine kubische Kurve von α abgebildet. Betrachten wir z.B. die Kurven k^3 durch $P_2, \ldots, P_6$, die $P_1 P_7$ schneiden. Die Verbindungsgeraden der nicht zu $P_1 P_7$ gehörenden Schnittpunkte dieser Kurven mit einer Ebene durch $P_1 P_7$ bilden den Strahlenbüschel um den nicht zu $P_1 P_7$ gehörenden Schnittpunkt dieser Ebene mit k_{17}^3. Die durch P_7 gehenden Bisekanten dieser Kurven k^3 bilden also den Kegel, der k_{17}^3 aus P_7 projiziert; das System Σ_{17} wird also abgebildet auf die Projektion $k_{17}'^3$ von k_{17}^3 aus P_7; diese ist eine ebene kubische Kurve, die in P_1' einen Doppelpunkt hat und durch $P_2', \ldots, P_6'$ geht.

§ 4. Die Kongruenz enthalt ∞^1 Kurven die einen Doppelpunkt haben. Diese sind biquadratische Kurven der ersten Art. Sie gehen auch durch den Punkt P_8, der mit $P_1, \ldots, P_7$ eine Gruppe von acht assoziierten Punkten bildet. Der Ort dieser Kurven ist eine Fläche der Ordnung 24, die in jedem der Punkte $P_1, \ldots, P_8$ einen zwölffachen Punkt hat. Dies zeigt sich, wenn wir den Durchschnitt dieser Fläche mit einer quadratischen Fläche durch $P_1, \ldots, P_8$ betrachten.

Die Doppelpunkte der Kurven dieses Systems Σ_d bilden den Ort der Spitzen der Kegel, die $P_1, \ldots, P_8$ enthalten, das ist eine Kurve sechster Ordnung k^6 vom Geschlechte drei, die jede der 28 Geraden $P_i P_8$ zweimal schneidet und durch die Punkte geht, worin die P_i enthaltende Bisekante von k_i^3 diese Kurve schneidet.

Das System Σ_d wird also abgebildet auf eine Kurve sechster Ordnung mit Doppelpunkten in $P_1', \ldots, P_6'$ und S'.

§ 5. Den Punkten K einer Gerade a von α entsprechen die Kegel $\varkappa$,

deren doppelte Erzeugenden in der Ebene P_7a liegen und die Flächen φ^2, welche die Ebene P_7a berühren. Sechs von den genannten Kegeln gehen durch einen willkürlichen Punkt Q, weil die *Jacobische* Kurve des Netzes der kubischen Kurven, das seine Basispunkte hat in $P'_1, \ldots, P'_6$ und in der Projektion Q' von Q, die Gerade a sechsmal schneidet. Und drei der genannten Flächen φ^2 gehen durch einen allgemeinen Punkt des Raumes.

Eine auf einen Punkt K von a abgebildete Kurve k^4 ist jedesmal Durchschnitt eines Kegels $\varkappa$ des angegebenen Systems und der Fläche φ^2, welche die Doppelgerade von $\varkappa$ enthält. Wenn wir diesen Kegel $\varkappa$ und diese Fläche φ^2 immer einander zuordnen, so bekommen wir eine eineindeutige Verwandtschaft zwischen den genannten Systemen von Kegeln $\varkappa$ und Flächen φ^2. Diese Verwandtschaft erzeugt eine Fläche von der Ordnung 21, die in $P_1, \ldots, P_6$ neunfache Punkte und in P_7 einen fünfzehnfachen Punkt hat. Die Ebene P_7a löst sich von dieser Fläche zweimal ab.

Die Kurven k^4, die auf die Punkte K einer Gerade a von α abgebildet werden, bilden also eine Flache neunzehnter Ordnung φ^{19}, die in den Punkten $P_1, \ldots, P_6$ neunfache Punkte, in P_7 einen dreizehnfachen Punkt und in P_8 einen sechsfachen Punkt besitzt.

§ 6. Weil eine Gerade l des Raumes 19 Kurven von φ^{19} schneidet, hat die Bildkurve des Systems Σ der Kurven k^4, die l schneiden, 19 Punkte mit einer Gerade a von α gemein. Wenn wir bemerken, dasz l drei Erzeugenden des Kegels $\varkappa_7$ und sieben Kurven k^4 jeder Fläche φ^7_i schneidet, so finden wir:

Die Kurven k^4, die eine Gerade l schneiden, worden abgebildet auf die Punkte K einer Kurve neunzehnter Ordnung k^{19}, die in S einen dreifachen Punkt und in den Punkten $P'_1, \ldots, P'_6$ siebenfache Punkte besitzt.

Ebenso zeigt sich:

Die Kurven k^4, die eine Gerade durch P_1 noch einmal schneiden, werden abgebildet auf die Punkte K einer Kurve zehnter Ordnung k'^{10}, die in S' und P'_1 Doppelpunkte hat und in $P'_2, \ldots, P'_6$ vierfache Punkte besitzt.

Die Kurven k^4, die eine Gerade durch P_7 noch einmal schneiden, werden abgebildet auf die Punkte K einer Kurve sechster Ordnung k'^6 die in $P'_1, \ldots, P'_6$ Doppelpunkte besitzt.

Die Kurven k^4, die eine Gerade durch P_8 noch einmal schneiden, werden abgebildet auf eine Kurve dreizehnter Ordnung k'^{13} die durch S geht und in den Punkten $P'_1, \ldots, P'_6$ fünffache Punkte besitzt.

§ 7. Eine Kurve k'^{19} schneidet eine andere Kurve derselben Art in 58 für die Abbilding nicht singulären Punkten.

Es gibt also 58 rationale biquadratische Raumkurven, die durch sieben gegebene Punkte gehen, und zwei gegebene Geraden schneiden.

Eine Kurve k'^{19} hat mit einer Kurve k'^{10} und mit einer Kurve k'^{6} dreissig, mit k'^{6}_{d} 24 für die Abbildung nicht singuläre Punkte gemein.

Die rationalen biquadratischen Kurven durch sieben gegebene Punkte, die eine gegebene Gerade schneiden, bilden also eine Flache 58^er Ordnung, die in jedem der gegebenen Punkte einen 28-fachen und in dem achten assoziierten Punkte einen 24-fachen Punkt besitzt.

Die sieben Kurven k^3_i sind dreifache Kurven der Fläche, weil jede von ihr mit drei Geraden durch P_i eine ausgeartete Kurve der Fläche bilden. Die 21 Geraden P_iP_k sind fünffache Geraden der Fläche.

Eine Kurve k'^{10} schneidet eine Kurve k'^{19}, eine demselben Punkte P zugeordnete Kurve k'^{10}, eine einem anderen Punkte P zugeordnete Kurve k'^{10}, eine Kurve k'^{6} und k'^{6}_{d} bzw. in 30, 12, 16, 16 und 12 Punkten, die für die Abbildung nicht singulär sind.

Hieraus ergibt sich:

Die Kurven k^4, die eine Gerade durch einen der gegebenen Punkte P noch einmal schneiden, bilden eine Fläche 30^er Ordnung, die in dem genannten Punkte P einen achtzehnfachen, in jedem der anderen gegebenen Punkte P einen vierzehnfachen und in P_8 einen zwölffachen Punkt besitzt.

Dasselbe folgt aus dem Umstande, dass eine Kurve k'^{6} eine Kurve k'^{19}, eine Kurve k'^{6}, eine Kurve k'^{10} und k'^{6}_{d} bzw. in 30, 12, 16 und 12 für die Abbildung nicht singulären Punkten schneidet.

§ 8. Wir betrachten eine biquadratische Kurve *erster* Art c^4 durch $P_1, \ldots, P_7$, die also auch den Punkt P_8 enthält. Auf jeder Fläche des durch c^4 bestimmten Büschels liegen zwei Kurven k^4 ohne Doppelpunkt.

Eine Kurve k^4 ohne Doppelpunkt, die sich auf einer Fläche des genannten Büschels befindet, schneidet eine der P_7 enthaltenden Erzeugenden dieser Fläche noch zweimal. Weil diese Erzeugende die Kurve c^4 noch einmal schneidet, werden die betrachteten Kurven eines $P_1, \ldots, P_7$ enthaltenden Büschels abgebildet auf die Punkte der projizierten Basiskurve, das ist eine kubische Kurve c'^3 durch $P'_1, \ldots, P'_6$ und durch die Projektion von P_8. Diese Projektion ist S', weil die Kurve k^3_7 die Gerade P_7P_8 zweimal schneidet.

Hieraus folgt, dass die genannten Kurven k^4 eine Fläche zwölfter Ordnung bilden, die in den Punkten $P_1, \ldots, P_7$ sechsfache Punkte hat und in P_8 einen vierfachen Punkt besitzt.

Die Kurve c^4 ist eine vierfache Kurve dieser Fläche. Diese Fläche ist nämlich der Ort der Kurven k^4, die c^4 noch in einem von $P_1, \ldots, P_7$ und auch von P_8 verschiedenen Punkte schneiden. Die Projektionen der Kurven k^4, die durch einen allgemeinen Punkt C von c^4 gehen, sind die nodalen Kurven k'^3, die durch die auf c'^3 sich befindende Projektion C' van C gehen und deren Doppelpunkte auf c'^3, sondern nicht in $P'_1, \ldots, P'_6$ oder in C' liegen.

Die Jacobische Kurve des von den Punkten $P'_1, \ldots, P'_6$ und C' be-

stimmten Netzes von kubischen Kurven ist sechster Ordnung und hat Doppelpunkte in $P'_1, \ldots, P'_6$ und C'. Sie schneidet c'^3 ausser in den letztgenannten Punkten noch viermal. Durch Q gehen also vier Kurven k^4. Weil Q ein allgemeiner Punkt des Raumes ist, haben wir gefunden, dass es vier rationale biquadratische Kurven gibt, die durch acht gegebene Punkte des Raumes gehen.

Physics. — *Measurements on the electrical resistance of some metals below the boiling point of oxygen.* By W. TUYN. (Communication N⁰. 196*b* of the Physical Laboratory at Leiden). (Communicated by Prof. W. H. KEESOM).

(Communicated at the meeting of February 23, 1929).

§ 1. When collaborating with professor KAMERLINGH ONNES in measuring the electrical resistance of some metals in liquid helium, I had the opportunity to measure their resistances in liquid oxygen and hydrogen also. The results may follow here, as accurate measurements of this kind are few in number; for the sake of completeness some data obtained at liquid helium temperatures are added [1]).

Cryostat, stirring apparatus and temperature regulator are of the usual Leiden type. The measuring method used is that of compensating the potential at the ends of the unknown and a known resistance, connected in series, with the aid of a DIESSELHORST compensation apparatus, free of thermo-forces, and supplied by O. WOLFF. The temperatures of the baths were measured and regulated with a platinum thermometer, allmost allways *Pt-23-1915* [2]), after the method of overlapping shunts according to KOHLRAUSCH; the platinum thermometer was calibrated with the helium gasthermometer. Zero-point determinations were made in the way, described allready before. [3])

As this paper must be regarded as a continuation on others, we may refer to the latter [4]) for details about the resistances, such as the purity of the material used and the manner in which they are constructed.

§ 2. *Cadmium.* The resistances are *Cd*-1919-*I* and *Cd*-1920-*I* and *II.* The material was obtained from KAHLBAUM, but the sample for

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[1]) Some of the data given in this paper have been used for composing Comm. Leiden Suppl. N⁰. 58, by W. TUYN and H. KAMERLINGH ONNES.

[2]) This thermometer, made in 1915, has been described by J. PALACIOS MARTINEZ and H. KAMERLINGH ONNES in Comm. Leiden N⁰. 156*b*; an interpolation table below —80° C. was given in Comm. Leiden Suppl. N⁰. 58.

[3]) H. KAMERLINGH ONNES and W. TUYN. Comm. Leiden N⁰. 160*a*.

[4]) H. KAMERLINGH ONNES and W. TUYN. Comm. Leiden N^os 160*a* and *b*; W. TUYN and H. KAMERLINGH ONNES. Comm. Leiden N^os 167*a* and 181.

the first resistance was the eldest. The two first resistances have been described and their different behaviour in liquid helium treated in an earlier paper.[1] Data are given in table I.

TABLE I. *Cd* — 1919 — *I.*
Cd — 1920 — *I.*
Cd — 1920 — *II.*

$\frac{T.}{^{\circ}K.}$	$(R/R_0)_{Cd-1919-I}$	$(R/R_0)_{Cd-1920-I}$	$(R/R_0)_{Cd-1920-II}$
90.41	0.28913	0.28820	
90.40			0.28907
81.02	0.25219		0.25218
81.01		0.25125	
73.06		0.21967	
73.05	0.22060		0.22061
65.99	0.19251	0.19158	0.19256
56.77	0.15572	0.15475	0.15581
20.51	0.02362	0.02267	0.02386
18.06	0.01759	0.01666	0.01786
16.53	0.01424	0.01331	0.01450
14.22	0.00997	0.00907	0.01022
4.22	0.00290	0.00143	
3.42	0.00237	0.00141	
2.81	0.00063		
2.62	supra-conductive		
1.46		0.00140	

Copper. Natural copper crystals were obtained from professor W. WIEN at Würzburg. In an other paper[1] particularities were given of the resistances, made from these crystals. The results of the measurements are given in table II.

[1]) W. TUYN and H. KAMERLINGH ONNES. Comm. Leiden N⁰. 181.

TABLE II. *Cu* — crystals — *I* — WIEN.
Cu — crystals — *III* — WIEN.
Cu — crystals — *IV* — WIEN.

$T.$ °K.	$(R/R_0)_{Cu-I}$	$(R/R_0)_{Cu-III}$	$(R/R_0)_{Cu-IV}$
88.56	0.1918	0.1749	0.1730
81.05	0.1578	0.1440	0.1407
73.11	0.1252	0.1103	0.1088
66.15	0.0966	0.0840	0.0821
55.00	0.0551	0.0469	0.0463
20.52	0.0042	0.0015	0.0018
18.03	0.0041	0.0013	0.0013
16.46	0.0035	0.0011	0.0012
14.32	0.0037	0.0010	0.0008

Gold. The wire *Au-12-1915*, described by CATH, KAMERLINGH ONNES and BURGERS [1]) as a drawn wire of 0.05 mm diameter, of pure minting gold, and also calibrated by them, was measured again in liquid oxygen and hydrogen. The calibration in liquid oxygen is the result of two series of measurements, a time of one year lying between both; the agreement is good. Data are given in table III.

TABLE III. *Au* — 12 — 1915.

$T.$ °K.	$(R/R_0)_{Au-12-1915}$	$T.$ °K.	$(R/R_0)_{Au-12-1915}$
90.41	0.27342	56.77	0.13031
88.57	0.26565	56.76	0.13037
81.05	0.23407		
81.02	0.23400	20.51	0.00912
73.06	0.20008	18.06	0.00656
73.06	0.20015	16.53	0.00539
65.99	0.16992	14.22	0.00415
65.98	0.16983		

1) Comm. Leiden N⁰. 152*c*.

Indium. From chemically pure indium, supplied by E. DE HAËN, G. m. b. H., the resistances *In*-1922-*I*,-*II*,-*III* and -*A* were made in the way, described allready before. [1]) The values of the resistances follow in table IV.

TABLE IV. *In*—1922—*I*. *In*—1922—*III*.
In—1922—*II*. *In*—1922—*A*.

$T.$ °K.	$(R/R_0)_{In-1922-I}$	$(R/R_0)_{In-1922-II}$	$(R/R_0)_{In-1922-III}$	$(R/R_0)_{In-1922-A}$
90.30	0.3708	0.28759	0.28763	0.28905
79.03	0.3361	0.24922	0.24923	0.25054
71.02	0.3115	0.22202	0.22202	0.22319
63.12	0.2873	0.19527	0.19525	0.19635
54.79	0.2618	0.16717	0.16717	0.16815
20.45	0.1602	0.05739	0.05737	0.05781
18.14	0.1548	0.05173	0.05174	0.05213
16.48		0.04796	0.04796	
16.47	0.1511			0.04834
14.20	0.1463	0.04317	0.04315	0.04352
4.22	0.1373		0.03390	0.03420
3.60	0.1372	0.03392		0.03418
3.42	0.1371	0.03387	0.03381	0.03392
3.40	0.1364	supra-conductive	supra-conductive	supra-conductive
3.38	supra-conductive			

Lead. The resistances *Pb*-1919-*I* (made of *Pb* "KAHLBAUM") and *Isotope Pb*-1919-*I* (*RaG*), described allready before [2]), were measured again May 1922, when placed in different baths. The temperature-resistance curve, for liquid hydrogen, does not wholly coincide with that found two years before [3]), the additive resistance having changed. Measurements are given in table V.

[1]) W. TUYN and H. KAMERLINGH ONNES. Comm. Leiden N⁰. 167*a*.
[2]) H. KAMERLINGH ONNES and W. TUYN. Comm. Leiden N⁰. 160*b*.
[3]) W. TUYN and H. KAMERLINGH ONNES. Comm. Leiden N⁰. 181.

TABLE V. *Pb*—1919—*I*.
Isotope Pb—1919—*I*.

$\frac{T.}{°K.}$	$(R/R_0)_{Pb-1919-I}$	$(R/R_0)_{Isotope\ Pb-1919-I}$	$\Delta\,(R/R_0)_{Pb-Isot.\ Pb}$
88.56	0.28947	0.28962	— 0.00015
81.05	0.26171	0.26178	7
73.11	0.23208	0.23218	10
66.15_5	0.20600	0.20617	17
55.00	0.16384	0.16397	13
20.52	0.03014	0.03042	28
18.02_5	0.02188	0.02215	27
16.46	0.01715	0.01742	27
14.32	0.01135	0.01164	29
7.2	supra-conductive	supra-conductive	

Platinum. As the platinum wire is the usual auxiliary thermometer,

TABLE VI. *Pt* — 1914 — *C*.
Pt — 1914 — *D*.

$\frac{T.}{°K.}$	$(R/R_0)_{Pt-1914-C}$	$(R/R_0)_{Pt-1914-D}$	$\Delta\,(R/R_0)_{C-D}$
88.57	0.25054	0.24894	0.00160
81.05		0.21680	
73.06		0.18275	
65.98	0.15490	0.15310	180
56.76	0.11771	0.11582	189
20.47	0.02082	0.01886	196
18.04	0.01893	0.01700	193
16.47	0.01798	0.01606	192
14.16	0.01696	0.01503	193
4.23	0.01559	0.01372	187
2.62	0.01556	0.01369	187
1.36		0.01370	

Data on this metal may be found in the publications of several laboratories. It therefore seems unnecessary at this moment to publish the calibrations of the platinum resistance thermometers, kept in the archives of the Leiden laboratory. We must however make an exception for the wires *Pt*-1914-*C* and -*D*, probably sent to Leiden by H. SCHULTZE of the P. T. R. at Charlottenburg, before the war. After a private communication of professor KAMERLINGH ONNES this was done to test them on supra-conductivity, for in that time it was thought possible that each metal could become supra-conductive when absolutely pure, and both wires were supposed to be of high purity. Both resistances have been described allready before. [1]) Data are given in table VI. (See p. 119).

Thallium. The resistances measured, made of thallium from KAHLBAUM, are *Tl*-1916-*VIII* and *Tl*-1916-*IX*, described in an earlier paper. [2]) Data are given in table VII.

TABLE VII. *Tl* — 1916 — *VIII*.
Tl — 1916 — *IX*.

$\frac{T.}{{}^{\circ}K.}$	$(R/R_0)_{Tl-1916-VIII}$	$(R/R_0)_{Tl-1916-IX}$	$\Delta (R/R_0)_{VIII-IX}$
88.57	0.27286	0.27205	+ 0.00081
81.05	0.24611	0.24529	82
73.06	0.21775	0.21692	83
65.98	0.19272	0.19187	85
56.76	0.16009	0.15929	80
20.47	0.03116	0.03023	93
18.04	0.02379	0.02286	93
16.47	0.01935	0.01841	94
14.16	0.01350	0.01259	91
4.23	0.00168	0.00084	84
3.26	0.00155	0.00072	83
2.62	0.00151	0.00069	82
2.48	0.00149	0.00066	83
2.47	supra-conductive	supra-conductive	

Tin. Four of the resistances *Sn*-1922-*I*, -*II*, -*III*, -*IV*, -*V* and -*VI*, all made of tin "KAHLBAUM", have been described before [1]); the remaining two are of the same type. Data are given in tables VIII and IX.

[1]) W. TUYN and H. KAMERLINGH ONNES. Comm. Leiden N⁰. 181.
[2]) H. KAMERLINGH ONNES and W. TUYN. Comm. Leiden N⁰. 160*a*.

TABLE VIII. *Sn — 1922 — I.*
Sn — 1922 — II.
Sn — 1922 — III.

$\frac{T.}{°K.}$	$(R/R_0)_{Sn-1922-I}$	$(R/R_0)_{Sn-1922-II}$	$(R/R_0)_{Sn-1922-III}$
90.01	0.26230	0.26039	0.25777
78.73	0.21723	0.21550	0.21311
71.02	0.18654	0.18485	0.18257
63.16	0.15499	0.15343	0.15146
57.49	0.13226	0.13087	0.12909
20.51	0.01283	0.01262	0.01236
17.96	0.00883	0.00868	0.00846
16.40	0.00680	0.00668	0.00653
13.98	0.00428	0.00420	0.00408
3.77	0.00079		
3.72	supra-conductive	supra-conductive	supra-conductive

TABLE IX. *Sn — 1922 — IV.*
Sn — 1922 — V.
Sn — 1922 — VI.

$\frac{T.}{°K.}$	$(R/R_0)_{Sn-1922-IV}$	$(R/R_0)_{Sn-1922-V}$	$(R/R_0)_{Sn-1922-VI}$
90.01	0.26064	0.25975	0.25966
78.73	0.21593	0.21503	0.21496
71.02	0.18550	0.18460	0.18448
63.16	0.15414	0.15327	0.15318
57.49	0.13157	0.13074	0.13066
20.51	0.01277	0.01270	0.01267
17.96	0.00878	0.00877	0.00875
16.40	0.00674	0.00675	0.00673
13.98	0.00423	0.00426	0.00425
3.77	0.00075	0.00082	0.00080
3.72	supra-conductive	supra-conductive	supra-conductive

TABLE X. *Sn* — single — crystal — 1922 — *VII*.
Sn — 1922 — *A*.

$\frac{T.}{°K.}$	$(R/R_0)_{Sn\text{—single crystal—}\text{—}1922\text{—}VII}$	$(R/R_0)_{Sn\text{—}1922\text{—}A}$	$\Delta (R/R_0)_{VII\text{—}A}$
90.30	0.2774	0.25449	0.0229
79.02	0.2292	0.20983	0.0194
71.02	0.1942	0.17792	0.0163
63.12	0.1559	0.14676	0.0091
54.79	0.1216	0.11450	0.0071
20.45	0.0133	0.01162	0.0017
18.14	0.0100	0.00836	0.0016
16.48	0.0082	0.00637	0.0018
14.20	0.0058	0.00409	0.0017
4.23		0.00100	
3.76		0.00099	
3.72		supra-conductive	

TABLE XI. *Zn*—1921— *I*.
Zn—1921—*II*.

$\frac{T.}{°K.}$	$(R/R_0)_{Zn\text{—}1921\text{—}I}$	$(R/R_0)_{Zn\text{—}1921\text{—}II}$	$\Delta (R/R_0)_{I\text{—}II}$
88.72	0.25367	0.25300	0.00067
81.04	0.22136	0.22078	58
72.71	0.18643	0.18581	62
66.10	0.15901	0.15838	63
57.86		0.12505	
57.83	0.12552		
20.48_5	0.01383	0.01278	0.00105
17.99	0.01119	0.01006	113
16.45	0.00984	0.00875	109
14.24	0.00838	0.00738	100
4.22	0.00378		
1.40	0.00378		

In order to obtain an idea about the behaviour of a single-crystal, regarding its electrical conductivity at low temperatures, a comparison was made between the single-crystal *Sn-single-crystal-*1922-*VII*, procured by the PHILIPS LAMPWORKS, and the extruded wire *Sn*-1922-*A*, made of tin "KAHLBAUM". Unfortunately the single-crystal was damaged, before measurements in liquid helium were made. The difference between the resistances in liquid oxygen diminishes here; that of two *wires* remains nearly constant on the other side. Data are given in table X.

Zinc. The resistance *Zn*-1921-*I*, made of *Zn* "KAHLBAUM", has been described allready before [1]; the resistance *Zn*-1921-*II* was made in the same way. Data are given in table XI.

I express my hearty thanks to Miss H. VAN DER HORST for her help in measuring and regulating the temperatures with the platinum thermometer.

[1]) W. TUYN and H. KAMERLINCH ONNES. Comm. Leiden N⁰. 181.

Histology. — *Ueber die Entstehung der kurzen Kontraktionswellen in der quergestreiften Muskulatur der Säugetiere.* (Vorläufige Mitteilung). Von B. I. LAWRENTJEW, Moskau. (Communicated by Prof. J. BOEKE).

(Communicated at the meeting of February 23, 1929).

Wie bekannt unterscheidet sich die Länge der Kontraktionswelle bei Einzelzuckung der quergestreiften Muskelfaser bei Wirbellosen und bei Wirbeltieren stark voneinander. Insbesondere nimmt man für Warmblüter an, dass die Länge der Kontraktionswelle grösser als die der Muskelfaser selbst ist. Umgekehrt ist die Wellenlänge bei einigen Insekten bei Einzelzuckung kürzer als die Länge der Muskelfaser (ROLLET) (4). In diesem Falle gelingt es, den Verlauf der Welle längs der Muskelfaser in Gestalt eines am Muskel sich fortbewegenden Knotens zu erfassen. Dieses interessante Phänomen hat es ermöglicht, die Morphologie der Kontraktionen an lebenden Muskelfasern zu erforschen (HÜRTHLE) (3).

In Alkohol fixierte Käfermuskeln ergeben ebenfalls das Phänomen einer kurzen Welle. In diesem Falle entspricht das Bild der Verdickung im Wellengebiete nicht in allem den Beobachtungen am lebenden Muskel, da die Menge der von der Welle erfassten Kommata bedeutend grösser als bei Kontraktion im lebenden Zustande ist. Es unterliegt jedoch keinem Zweifel, dass das Phänomen der kurzen Welle auf dem fixierten Präparate eine Erscheinung derselben Ordnung wie bei der Kontraktion im lebenden Zustande darstellt. Man nimmt an, dass die kurzen Wellen eines fixierten Muskels das Resultat einer gewissen Summierung mehrerer kurzen Wellen sind.

Was nun die quergestreiften Muskeln der Säugetiere anbetrifft, so lassen sich hier auf den fixierten Präparaten häufig Kontraktionsknoten beobachten, deren Entstehung und Charakter jedoch, wie HEIDENHAIN (2) mit Recht hervorhebt, keine Gesetzmässigkeit zeigt und nicht den Typus wirklicher Wellen besitzt.

Bei Untersuchung der Speiseröhre eines Hundes ist mir jedoch die Feststellung des Phänomens der kurzen Welle mit Anzeichen einer bestimmten Gesetzmässigkeit gelungen, weshalb ich es für nicht uninteressant halte, darüber zu berichten.

Wie bekannt, besteht die muscularis propria der Speiseröhre eines Hundes bis zur cardia hauptsächlich aus quergestreiften Muskelfasern.

Durch Imprägnation nach BIELSCHOWSKY lässt sich leicht nachweisen, dass jede quergestreifte Muskelfaser durch den motorischen Apparat von den markhaltigen Nervenfasern versorgt wird. Diese Verhältnisse hat SABUSSOW (5) bei seinen Arbeiten nach der Methylenblaumethode vorzüglich nachgewiesen. Die Nervenendigungen zeigen den Charakter der für die quergestreifte Muskulatur typischen motorischen Endplatte. Sehr häufig tritt die Endplatte über das allgemeine Niveau der Muskelfaser hervor und bildet eine Art von Hugel. Die Sohlenplatte enthält 5—12 Kerne. Bei gelungener Imprägnation tritt das von J. BOEKE (1) entdeckte und ins Sarkoplasma der Muskelfaser übergehende periterminale Netzwerk sehr deutlich hervor [1]).

Durch ein- und beiderseitige Durchschneidung der n.n. vagi und nachfolgende Untersuchung des Degenerationsverlaufs der Nervenfasern in der Wand der Speiseröhre ist es mir gelungen, festzustellen, dass die markhaltigen Nervenfasern, die auf den quergestreiften Muskeln der Speiseröhre motorische Endplatten ergeben, den n. vagi angehören.

Es wird also die Innervation der quergestreiften Muskeln der Speiseröhre eines Hundes bis zur cardia ohne Unterbrechung der zentralen Fasern in den peripheren Ganglien vorgenommen.

Die motorischen Fasern der n. vagi und ihre Endigungen in den quergestreiften Muskeln degenerieren am 5.—6. Tage nach Durchschneidung der n. vagi. Am 7. Tage nach Durchschneidung des rechten n. vagus fand ich, dass die vom linken n. vagus innervierten quergestreiften Muskelfasern (d.h. die Fasern, deren motorische Apparate intakt geblieben waren) typische kurze Kontraktionswellen besassen.

Das Bild einer solchen kurzen Welle ist, wie das aus den beigefügten Abbildungen gut hervorgeht, ein derart charakteristisches, dass man es in keinem Falle mit den Kontraktionsknoten, wie sie in fixierten Muskeln angetroffen werden und die in den meisten Fällen Absterbeerscheinungen der Muskelfaser darstellen, verwechseln darf.

Die von mir wahrgenommenen kurzen Wellen befinden sich in allen Fällen auf dem Niveau der motorischen Endplatte, erstrecken sich in völlig gleicher Entfernung zu beiden Seiten des Endplattenzentrums und gehen allmählich in normale Querstreifen über. Diese Wellen zeigen demnach dasselbe Bild, wie es an Käfermuskeln erhalten worden ist.

Die Menge der von der Welle erfassten Kommata schwankt in den Grenzen von 12—60. Die Abbildungen 1,3. geben eine anschauliche Vorstellung von diesem Phänomen. Hier ist es uns gelungen, gleichzeitig

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[1]) Die Präparate aller hier beschriebenen Versuche wurden in folgender Weise hergestellt: Lebensfrische Stückchen der Speiseröhre wurden mit der Muscularis nach oben auf Paraffinplättchen gespannt und in folgendem Gemisch fixiert: Ac. arsenicosi 1%—30.0, Alcohol aeth. rect. 96°—30.0, Formol —30,0. Hier bleiben die Stückchen 1 Stunde. Im weiteren wurden die Stückchen ohne vorherige Spülung in 20 prozentiges Formol gebracht, in dem sie ohne Schaden längere Zeit bleiben können. Gefrierschnitten. Nervenfärbung nach BIELSCHOWSKY-GROS, Vergoldung. Hämatoxilin. Einbettung in Lävulose nach HERINGA.

auch die motorische Endplatte mit der Nervenfaser zu färben und daher tritt der Zusammenhang der Welle mit dem Nervenapparate besonders deutlich hervor.

Meine Aufgabe bestand in der unwiederleglichen Feststellung folgender vier Punkte: 1. Entstehen die kurzen Wellen in Wirklichkeit auf Muskelfasern mit intakt gebliebenen Nervenapparaten. 2. Ist die Entstehung kurzer Wellen auf Fasern mit degenerierten Nervenendigungen möglich. 3. Zu welchen Fristen nach der Operation tritt das Phänomen der kurzen Welle am deutlichsten hervor. 4. Lassen sich kurze Wellen bei normalen, nicht operierten Tieren beobachten.

Zu diesem Zwecke durchschnitt ich die n. vagi auf der einen Seite des Halses, nahm eine beiderseitige Vagotomie vor, tötete nach Verlauf verschiedener Fristen die operierten Tiere und untersuchte endlich die Speiseröhren normaler, nicht operierter Tiere. Bis jetzt ist mir folgendes festzustellen gelungen:

1. Muskelfasern, die ihre Nervenapparaten von einem unversehrten n. vagus erhalten, ergeben vom 7.—14. Tage nach Durchschneidung des anderen n. vagus am besten ausgeprägte Wellen.

2. Angefangen vom 2. Tage nach Ausschaltung des die gegebenen Muskelfasern innervierenden n. vagus lassen sich keine kurzen Wellen auf ihnen beobachten, d. h. die denervierte Muskelfaser zeigt nicht das Phänomen der kurzen Wellen. Die Abbildungen 2 und 3 illustrieren die angeführten Verhältnisse. Beide Abbildungen sind ein und demselben Versuche und der gleichen Präparatserie entnommen. In dem einen Falle (Abb. 2) hat die denervierte Muskelfaser (14. Tag nach Ausschaltung des rechten n. vagus) eine völlig gleichartige, normale Form; im anderen Falle (Abb. 3) zeigen drei nebeneinander liegende Fasern mit intakt gebliebenen Nervenapparaten deutliche Kontraktionswellen. Endlich ist es mir noch gelungen, kurze Wellen auf den Muskelfasern einer von normalen, nicht operierten Hunden genommenen Speiseröhre zu erhalten. Jedoch werden diese Wellen hier nicht so häufig beobachtet und sind nicht so gut ausgeprägt.

Hier im Gebiete der Endplatten lässt sich bisweilen eine für die Welle typische Anschwellung und Annäherung der Menisken beobachten, jedoch keine so intensive, wie bei den nach Vagotomie beschriebenen Fällen.

Auf Grund des bis jetzt erhaltenen Materials habe ich natürlich zu irgendeiner Schlussfolgerung, inwieweit das beobachtete Phänomen den im lebenden Organismus bestehenden physiologischen Kontraktionsbedingungen entspricht, kein Recht. Es ist durchaus möglich, dass die kurze Welle in der von mir im fixierten Präparate beobachteten Form nicht bei normaler Arbeit der Speiseröhrenmuskeln entsteht, was jedoch das Interesse an unserer Beobachtung nicht schmälert.

In erster Linie ist der Zusammenhang der kurzen Welle mit dem Nervenapparate von Interesse. Martin Heidenhain suchte das Wesen der kurzen Wellen in der Struktur der kontraktilen Substanz selbst. Die

Phylogenese der quergestreiften Muskeln geht nach M. HEIDENHAIN in der Richtung einer Zunahme der Wellenlänge vor sich. Aus diesem Grunde haben die "unvollkommenen" Käfermuskeln, die nur kurze Wellen ergeben können, eine Menge Innervationspunkte, "um die Homogenität der Funktion zu sichern".

In unserem Falle sehen wir jedoch ein unmittelbares Abhängigkeitsverhältnis der kurzen Welle vom Nervenleiter. Der denervierte Muskel ergibt kein Phänomen der kurzen Welle. Ausserordentlich interessant ist auch die Abhängigkeit, die zwischen den kurzen Wellen und der Durchschneidung eines der n. vagi besteht.

Man erhält den Eindruck, dass die Durchschneidung des einen n. vagus gewisse funktionelle Veränderungen beim intakt gebliebenen Partner nach sich zieht. In jüngster Zeit begegnen wir in der physiologischen Literatur Hinweisen, dass bei Durchschneidung eines der paarigen Nerven die Erregbarkeit des anderen in der Tat gesteigert wird.

Die erhaltenen Resultate gestatte ich mir in folgender Weise zusammenzufassen:

1. Die quergestreifte Muskulatur der Speiseröhre eines Hundes erhält vom n. vagus motorische Fasern, die sich ohne Unterbrechung in den peripheren Ganglien unmittelbar zu den Muskelfasern hinziehen.
2. Die Nervenapparate auf den quergestreiften Muskelfasern der Speiseröhre stellen typische für die willkürliche Muskulatur motorische Endplatten mit gut ausgeprägtem periterminalem Netzwerk dar.
3. Auf den quergestreiften Muskelfasern der Speiseröhre lässt sich in fixiertem Zustande das Phänomen der kurzen Kontraktionswelle beobachten, das den bei Käfermuskeln beschriebenen kurzen Wellen ausserordentlich ähnlich ist.
4. Auf den mit einem nicht degenerierten n. vagus in Zusammenhang stehenden Muskelfasern werden die kurzen Wellen bei Durchschneidung des anderen n. vagus mit der grössten Deutlichkeit vom 7. bis zum 14. Tage nach der Operation beobachtet.
5. Denervierte Muskelfasern der Speiseröhre ergeben keine kurzen Wellen.

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ERKLÄRUNG DER ABBILDUNGEN.

Abb. 1. Hund. Kurze Kontraktionswelle in der Muskelfaser der Speiseröhre. Die motorische Nervenendplatte nach Bielschowsky inprägniert. Photo.

Abb. 2. Hund. 14 Tage nach Durchscheidung des rechten n. vagus. Die Nervenfasern des rechten n. vagus und die von ihnen gebildeten Nervenendplatten völlig degeneriert. Keine Kontraktionswellen. Bielschowsky. Vergoldung. Hämatoxylin.

Abb. 3. Hund. 14 Tage nach Durchschneidung des rechten n. vagus. Kurze Kontraktionswellen in den Muskelfasern der Speiseröhre. Die Nervenfasern des linken vagus und ihre Endigungen intakt geblieben. Bielschowsky. Vergoldung. Hämatoxylin.

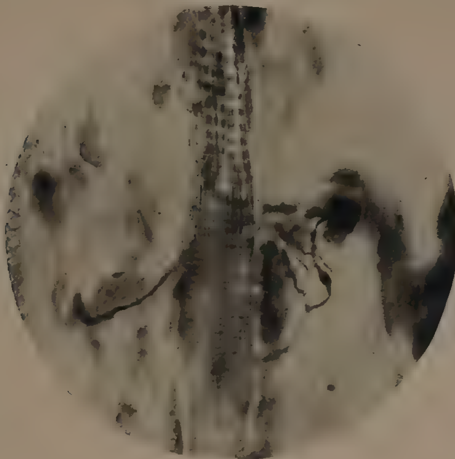


Fig. 1.

Fig. 2.

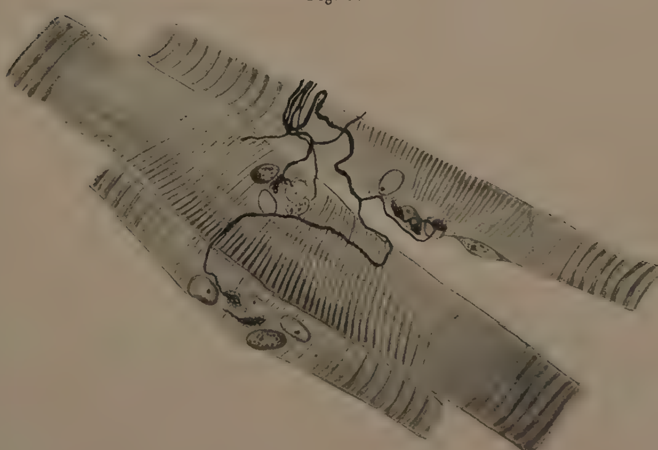


Fig. 3.

Histology. — *The microscopic Structure of neurone junctions in the central nervous System.* By O. W. TIEGS (Sidney). (Communicated by Prof. J. BOEKE.)

(Communicated at the meeting of February 23, 1929).

In the present paper are reported the results of observations [1]) on the microscopic structure of the zone of stimulus transference from neurone to neurone, made on a variety of nerve cells from the central nervous system.

As is well known, two opposed views have been expressed on this problem. On the one hand it is held that the neurofibrils of the two neurones in question are connected with one another; this is the doctrine of neurofibril continuity, developed especially by APATHY and BETHE. On the other hand it is maintained that no such connection exists (discontinuity), and to the occurrence of this break in the protoplasmic chain are attributed those features of reflex conduction, which distinguish it from conduction in the nerve fibre.

The doctrine of discontinuity arose in the observations of FOREL that in GOLGI preparations the finest nerve ramifications ended blindly; since the dendrites of nerve cells similarly ended blindly it was inferred that the „Endbäumchen" were the nerve terminals that ramified among the dendrites and on the cell-body of unimpregnated nerve cells. With the discovery of the silver methods of CAJAL and of BIELSCHOWSKY it was found that the nerve terminals were exceedingly closely applied to the surface of the nerve cells. CAJAL, throughout his great researches, has upheld the theory that the neurones remain distinct; there may be very close contact between them, but nevertheless they remain visibly separated. When stainable material is observed between the nerve terminals and the surface of the nerve cell, it is unhesitatingly described as adhesive „Kittsubstanz", without regard to the fact that it would be difficult to determine the adhesive nature of the structure by microscopic examination alone.

M. HEIDENHAIN has made the important observation that in spinal cord cells the nerve terminals may transverse the artificial shrinkage space that is produced by the action of the reagents on the tissue: there occurs, he believes, an intimate adhesion between the terminals and the nerve cell though not a fusion of their substance. But it might well be asked how one can distinguish by microscopic examination an „intimate adhesion" from a fusion

[1]) This work was carried out while holding a Rockefeller travelling fellowship in the Anatomical Laboratory Cambridge, and the Laboratory of Histology and Embryology, Utrecht. To Professors J. T. WILSON and J. BOEKE I wish to offer my thanks for the hospitality afforded me.

of protoplasm. BIELSCHOWSKY, WOLFF, HELD and others maintain finally, that neurofibrils from the terminals pass into the nerve cell and are continuous with its neurofibrils. This theory of neurofibril continuity has no opinion to offer as to whether there is an adhesion or a fusion between the neurofibrils; it merely asserts that so far as the microscope can reveal, the fibrillae exhibit no visible break at the region of stimulus transference.

The following observations have been made on material prepared by the silver methods of BIELSCHOWSKY and of CAJAL.

1. MAUTHNER's *Cells*. These are a single pair of very large nerve cells situated in the medulla oblongata of fishes and tailed amphibians, and related to the vestibular nerve and to other fibres subserving equilibration. According to BARTELMEZ (Journal compar. Neurology 25, (1915)) these fibres end in large numbers by minute end-knobs on the surface of the cell. MARUI (Journ. Comp. Neurology 30, (1918)) on the contrary finds that the fibrils actually penetrate into the substance of the cell.

In addition to these endings there occur, according to BARTELMEZ, certain large very coarse nerve terminals that end upon the giant lateral dendrite of the cell; they are said to end in close contact with the surface of the dendrite, but to remain perfectly distinct from it, and by proper cytological methods it is even possible to demonstrate a plasma membrane at their free ending. This is regarded by the author as a critical example of discontinuity, derived from the study of a case where the size of the object makes reliable observation possible.

The following observations have been made on material from the trout. In BIELSCHOWSKY preparations a dense mass of thick fibres encloses the body of the nerve cell; in places where the section has been suitably cut these coarse fibres may be observed to taper gradually into a thin filament of the greatest tenuity, and approach the surface of the cell. Now if, as is almost always the case, the neurofibril impregnation in the MAUTHNER's cell has failed, these fine threads end short on the surface of the cell: this is discontinuity. But when preparations are obtained where the fibrils have been rendered visible, it is at once observed that the intracellular fibrils are a direct continuation of the fine extracellular threads (Fig. 1.) In CAJAL preparations these terminals may present (but not always) minute rounded endings; I am unable to state what causes their appearance or non-appearance in various preparations. It may be added that in the CAJAL preparations even when these minute rounded ,,endbulbs" appear, fibrillae may, if the impregnation has succeeded, be observed passing over into the substance of the nerve cell.

In BIELSCHOWSKY preparations it is sometimes also possible to reveal a plexus of very delicate fibrils lying upon the surface of the dendrites and terminating upon these by typical flattened ,,end-feet"; but in the trout I have never observed anything resembling the very coarse endings described

by BARTELMEZ from Siluroid material. Concerning these coarse endings attention may be drawn to a serious difficulty in the author's evidence for their existence. It will be observed in his single illustration that they all terminate along the cut edge of the dendrite: they „end" therefore on the surface of the microtome section, and are therefore probably nothing but the thick nerve fibres that run in large numbers along the dendrite and have been severed by the microtome knife.

But further discussion may be reserved for a later paper.

2. *The connection between* MAUTHNER's *fibre and cells of the Spinal Cord in Amblystoma.*

The axons of the two Mauthner cells cross in the medulla to the opposite side and enter the spinal cord; here they may be observed in transverse sections as two thick fibres situated in the ventral white matter. At intervals they give off short, usually unbranched, thick truncated collaterals, sometimes scarcely longer than the diameter of the main fibre, which convey the stimulus to the large spinal cord cells. It is so easy to identify these endings and they are so exceedingly coarse, that they constitute a unique object upon which the zone of stimulus transference may be investigated.

The collaterals are applied to the dendrites of the spinal cord cells; sometimes the dendrite concerned may be long and very thin (fig. 10), at other times short and stout. In a small proportion of cases there is definitely no visible connection between the two; this is discontinuity (fig. 8.). But when the impregnation is more successful a thick protoplasmic strand that stains weakly may be seen joining the terminal to the dendrite. Such a connection has already been described by BECCARI (Arch. Ital. di Anat. e di Embr. 1918, Vol. 17). But in the best impregnations there may be seen with the utmost clearness a system of fibrils that traverse this „transitional substance" and are visibly continuous with the fibrils of the dendrite. Whether these fibrils and the „transitional substance" are derived from the nerve cell or the collateral I am not able to state; but of the existence of a perfectly continuous neurofibril path there can, in this case, be no question (see fig. 9.). Occasionally cases are found where the "transitional substance" is not visible, but only the neurofibrils (Fig. 10.).

3. *The connection between the vestibular nerve and cells of the tangential nucleus in the trout.*

Coarse fibres from the eighth cranial nerve on entering the brain come into association, by a short lateral thickening, with the rounded cells of the tangential nucleus; occasionally finer branches of this curious terminal may run over the surface of the cell.

When good BIELSCHOWSKY preparations are obtained the connection is well adapted for studying the intimate structure of the neurone junction. In such preparations I find that the neurofibrils pass into visibly direct continuity with those of the nerve cell (Fig. 2.).

Although it is very difficult to exclude fusion of intracellular fibrils, yet it seems that the fibrils do not form an intracellular network, as they do in certain other cells, but pass directly into the cell axon (Fig. 3.).

4. *Cells of the trapezoid nucleus (adult cat).*

Fibres of the acoustic tract terminate in these cells by the very characteristic calices of HELD. According to CAJAL these terminals, which clasp the cell firmly, remain distinct from it: HELD and later BIELSCHOWSKY (LEWANDOWSKY's Handbuch, Vol. 1) find, on the contrary, that the intracellular fibrils are intimately connected with the nerve terminals.

In BIELSCHOWSKY preparations from the trapezoid necleus of the adult cat I find that an obvious discontinuity such as CAJAL describes is to be seen only when the intracellular neurofibril impregnation fails. When, however, a good neurofibril impregnation is obtained it may prove exceedingly difficult to determine whether there is a continuous fibril path from the terminal to the nerve cell, or whether the two fibril systems remain distinct. Often the intracellular fibrillation may be so complex that it is impossible to make precise observations on it; but at other times the condition is much simpler. It may then be observed, sometimes with marked clearness, that the neurofibrillae of the nerve cell are directly joined to the pericellular terminal (fig. 4). A particularly conclusive example is illustrated in fig. 5. Here the cell and a branch of the terminal calyx have been cut in transverse section; the direct connection between the intracellular fibrils and the nerve terminal is obvious.

5. PURKINJE *cells of the cerebellum.*

On these remarkable cells are found various types of nerve terminals; two only wil be described here, the clasping endings of the "basket fibres", and the climbing tendril fibres.

Investigation of these cells by present methods is, in my experience unsatisfactory, for it is only in exceptional cases that the neurofibrillae appear with a clearness sufficient for precise histological observation. Under such condition discontinuity will be demonstrable in most of the cells examined. BIELSCHOWSKY and WOLFF (Journ. f. Psych. u. Neurol. 1904, Vol. 4) and especially clearly OUDENDAL (Psych. en Neurol. Bladen, 1912) find that in good impregnations the discontinuity between the clasping fibres and the body of the cell no longer occurs.

After a careful examination of preparations from the rabbits cerebellum kindly placed at my disposal by Professor BOEKE I have found a number of cases that clearly confirm OUDENDAL's description. In the great majority of the cells no trace of such connection, or even of sharply impregnated intracellular fibrillae is to be seen; but when the fibrillae become visible, it is seen at the same time, if true profile cases are examined, that the fibrils are directly joined to the pericellular ending (fig. 6). But in the case of the climbing tendril fibres, in my experience no such connection occurs; so far

as I have been able to observe the climbing fibrils may spread over the great dendritic formation of the PURKINJE cells, without the passing of any fibrils into its substance. Perhaps better methods will in the future reveal such a connection; but it is quite conceivable that no such connection occurs. For it seems justifiable to conclude that two such endings as, on the one hand, the delicate filaments that establish connection with MAUTHNER's cell, and on the other the tendril fibre that spreads over the whole surface of the cell, must have a fundamentally different mode of functioning. It is in fact probable that these peculiar endings constitute a genuine instance of a discontinuous-"parallel-contact" in HEIDENHAIN's use of that term.

6. *Cells of the spinal cord.*

In BIELSCHOWSKY preparations of the spinal cord of adult mammals the terminal "end-feet" of AUERBACH and HELD may be seen, sometimes in considerable numbers. They occur on the surface of the body of the cell and on the proximal part of the dendrites. Their investigation is in my experience most difficult. Discontinuity as CAJAL has described it may readily be seen; but such discontinuity appears to be merely a failure to demonstrate continuity. HEIDENHAIN (Plasma und Zelle) finds that they are finely attached to the surface of the cell, but denies that a neurofibrillar continuity occurs. BIELSCHOWSKY (in LEWANDOWSKY's Handbuch der Neurologie) finds that the neurofibrils merge into those of the nerve cells.

My observations have been made on very successful BIELSCHOWSKY preparations of the cord of an adult cat. When the "end-feet" are examined in true profile it is very often possible to see, even with considerable clearness, that one or several delicate fibrils pass from the ending on to the nerve cell, and in doing so frequently traverse the weakly staining so-called adhesive "Kittsubstanz". But it is usually exceedingly difficult to be certain, in such cases whether the fine fibrils actually enter the cell to join its fibrils, or merely end on its surface. The observation is more difficult even in thin sections, than one might expect: but in examining numerous such endings I have frequent examples which appear to me to be perfectly critical; true profile cases alone can of course be considered, but in such cases these most delicate fibrils can be observed to enter the substance of the cell. One such case is shown in fig. 7. With material which is so exceedingly difficult to prepare and to examine, it is understandable that sharp difference of opinion should exist on this question; but if the unique case of the termination of the MAUTHNER fibre collaterals can serve as an analogy, then obvious discontinuity at the surface of the "end-feet" may be regarded as due merely to the great difficulty of demonstrating this exceedingly delicate connection between the two neurones. In analogy it might further be suggested that the "Kittsubstanz" for whose adhesive nature there is really no direct evidence, is of the same nature as the weakly staining "transitional substance" between the MAUTHNER collaterals and the substance of the dendrite in the cord of Amblystoma.

It remains to draw attention to one important feature of spinal cord cells ; by far the greater number of neurofibrils enter the body of the cell through its dendrites, and have no direct relation to the "end-feet". In the cells of newly born animals in fact "end-feet" do not seem to occur. Here the neurone junctions must be established via the cell dendrites, which spread over a very extensive area of the gray matter. When we trace the dendrites of these cells outwards they are seen repeatedly to divide until the branches contain only a single neurofibril ; usually as we trace them further from the cell, they pass out of the plane of the microtome section ; but in very fortunate cases I have been able to observe that they are actually connected with collaterals that enter the gray matter from the white matter (Australian Journ. of Exp. Biol. and med. Sc. 1927, Vol. 4).

The connection between MAUTHNER's collateral and the dendrites of spinal cord cells in Amblystoma provides a case where this type of connection may be exhibited with ease and certainty. As the animal grows older the complexity of the cord increases enormously ; "end-feet" now also become visible. Nevertheless by far the greater number of neurofibrils enter the body of the cell through its dendrites : on account of the vast complexity of the tissue it is practically impossible to demonstrate the nature of the neurone junctions in the more peripheral regions of the dendritic expansion, but there seems no reason to believe that the condition of the young animal should be departed from.

REFERENCE TO FIGURES.

Fig. 1. Mauthner cell of young trout (BIELSCHOWSKY preparation: Obj. 3 mm. oe. $\times$ 15). The intracellular fibrils are impregnated only in places. Those of the large axon (a) are well shown. The cell is surrounded by a dense mass of nerve fibres, but only those at the lower part of the cell are true nerve terminals. They taper out into delicate threads and at least ten of them pass into the protoplasma of the cell. In the region where they do not pass in the neurofibril staining has failed.

Fig. 2. Cell of tangential nucleus of young trout (BIELSCHOWSKY Obj. 3 mm. oc. $\times$ 15). From the swollen terminal several fibrils pass into the substance of the cell; the figure cannot make it clear that the fibrils are not on the surface of the cell.

Fig. 3. The same from nearly hatched trout (Cajal preparation. Ob. 3 mm. oc. $\times$ 15). The fibrils pass from the terminal through the substance of the cell into the axon (a).

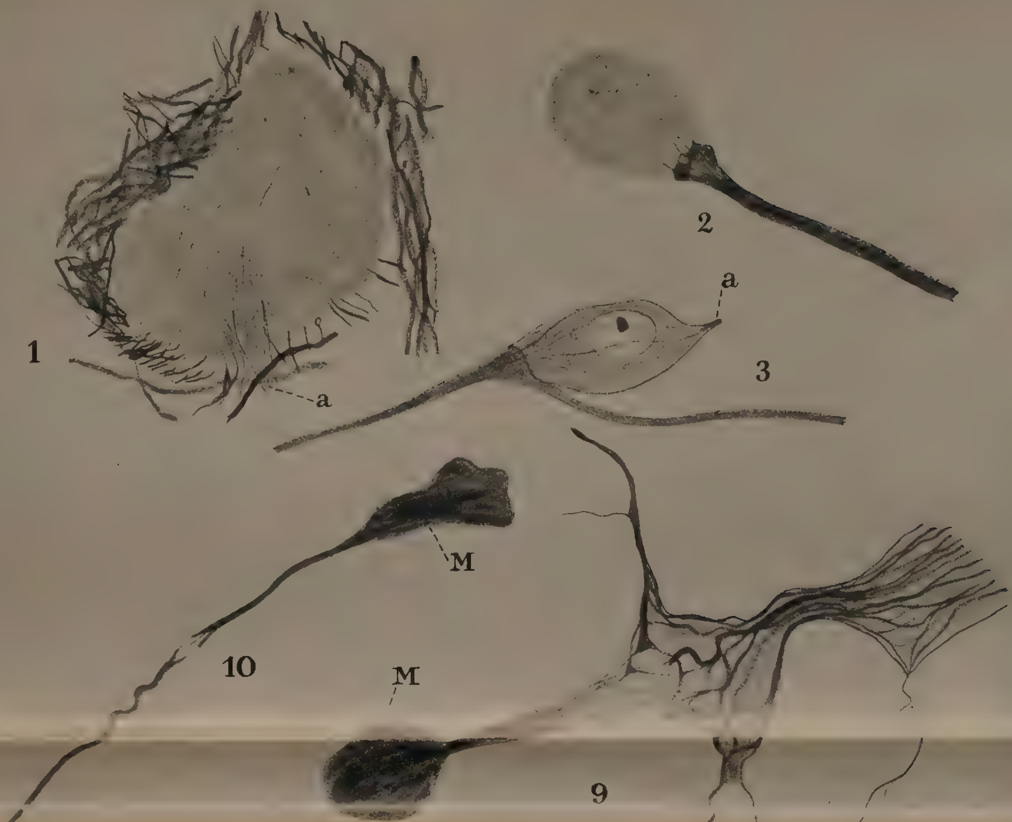
Fig. 4. Cell from trapezoid nucleus of adult cat. (BIELSCHOWSKY Obj. 3 mm. $\times$ 15). With the terminal calyx of HELD numerous fibrils are connected and pass upwards into the cell axon (a).

Fig. 5. The same, seen in a transverse section. From the coarse pericellular terminal fine fibrils are seen passing into the substance of the cell (BIELSCHOWSKY Obj. 3 mm. Oc. $\times$ 15).

6. Termination of basket fibres on body of Purkinje cell of rabbit. (BIELSCHOWSKY Obj. 3 mm. oc. $\times$ 15).

7. A terminal "end-foot" of Held on surface of spinal cord cell, showing passage of neurofibrils into the substance of the cell. (Cat. BIELSCHOWSKY Obj. 3 mm. oc. $\times$ 15).

Figs. 8, 9, 10. The connection between collaterals of MAUTHNER's fibre (M) and dendrites of spinal cord cells (amblystoma larva). In fig. 8 discontinuity is seen; in fig. 9 the more complete staining shows a complex fibril system passing across the junctional tissue connecting the collateral with the fibrils of the dendrite; in fig. 10 an exceedingly simple connection is seen. (BIELSCHOWSKY Obj. 3 mm. Oc. $\times$ 15).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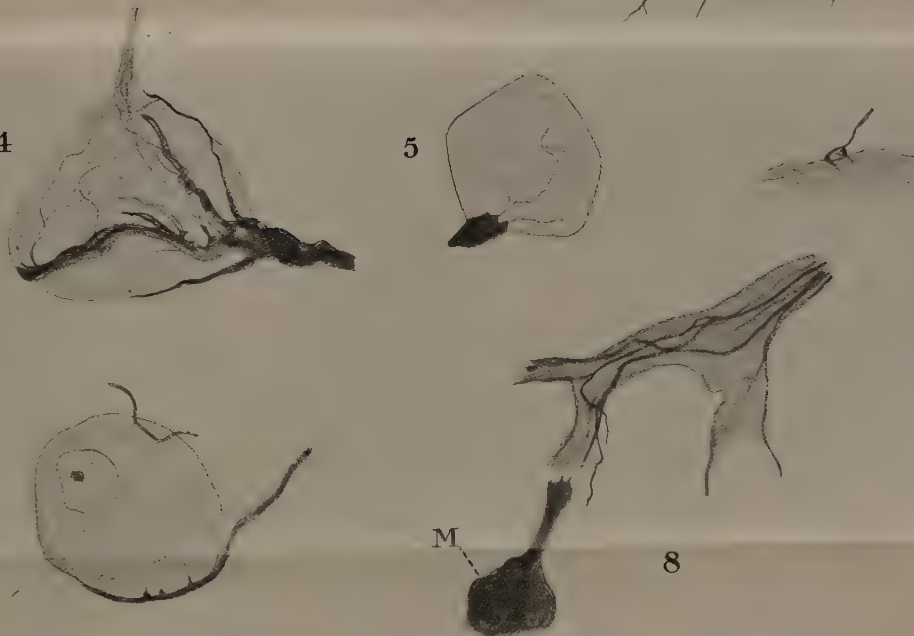

Botany. — *Influence of the nitrate-ion-concentration of nutrient solutions on the growth of summer-wheat.* By M. A. J. GOEDEWAAGEN. (Communicated by Prof. F. A. F. C. WENT.)

(Communicated at the meeting of January 26, 1929).

1. In this treatise the results are communicated of water-culture-experiments on summer-wheat, carried out for the purpose of investigating the influence of NO_3-ion-concentration on the growth. In those experiments we have restricted ourselves to those nitrate-concentrations of which it may be assumed that they may occur in the soil-solution, found in the ground among the soil-particles.

For years together several investigators have been occupied with the study of the soil-solution and numerous attempts have been made to isolate this solution in an unaltered condition. Beside this the extraction-method has frequently been applied, in which soil was extracted with pure water and from the composition of the extract found conclusions were drawn concerning the composition of the soil-solution. Among the most important recent investigations on this subject should be mentioned those by BURD and MARTIN [1] [2] and by PARKER [3]. In all probability these investigators have succeeded by application of the so-called displacement-method in isolating the soil-solution in its natural condition.

In the literature on soil-science we have consulted some analyses of soil-solutions, especially BURD and MARTIN's, in order to get an idea of the presumable range within which the nitrate-concentration in the soil-solution moves. This range appeared to be rather extensive. BURD and MARTIN [2] examined soils received from dry regions and belonging to one and the same type ("fine, sandy loam"), but of divergent fertility. As highest nitrate-nitrogen-concentration we found mentioned 493 p. p. m. The highest total salt-concentration was 4093 p. p. m. The authors state that these concentrations probably represent maximal values, which will as a rule no be reached in soils from moist regions.

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1) J. S. BURD and J. C. MARTIN (Univ. of Cal. Agric. Exp. Stat.) Water-displacement of soils and the soil solution. Journ. of agr. Sci. Vol. 13. 1923. p. 265—295.

2) J. S. BURD and J. C. MARTIN (Univ. of Cal. Agric. Exp. Stat.) Secular and seasonal changes in the soil solution. Soil science Vol. 18. 1924. p. 151—167.

3) F. W. PARKER (Agr. Exp. Stat. Univ. of Wisconsin) Methods of studying the concentration and composition of the soil-solution. Soil science Vol. 12. 1921. p. 209—232.

In connection with this we have chosen as highest nitrogen-concentration for our experiments circa 450 p. p. m. The highest total concentration in our experiments was 4125 p. p. m. The lower limit of the range of concentration examined by us we have put very low for nitrogen, viz. at 1.5 p. p. m., for in the soil-solution very slight nitrogen-concentrations may occur. HOAGLAND, BURD and MARTIN [4] [5] [6] found that the initially comparatively high nitrate-concentration in the soil-solution of cultivated soils is reduced to a very low minimum in the 5th to 8th week of the growth of the crop. Further it has been stated by PARKER and PIERRE [7] [8] in a study on the influence of low phosphate- and potassium-concentrations on the growth of barley and soybeans, that the "minimum concentration for satisfactory growth" of these ions is very low (respectively 0.1 and lower than 2 p.p.m.), so that in analogy with this a low minimum-concentration could also be expected for the nitrogen.

In this article four experiments are described, all of them carried out during the summermonths of 1928. The first three experiments relate to the influence of moderate and high nitrate-concentrations, the last to that of the low concentrations.

2. In the first weeks of their growth the plants were supplied with new solution every 5 to 6 days, later every 4 to 5 days. In the 2nd to the 4th experiments the nitrate-content of the solutions was determined at each renewal, both of the fresh-prepared solutions and of those with which the plants had been in touch for about 5 days. In the course of vegetation normally growing plants appeared averagely to absorb the following quantities of nitrogen per 24 hours in milligrams:

TABLE 1. Average daily N. absorption per plant from week to week in mgrs.

1st week	2nd week	3rd week	4th week	5th week	6th week	7th week
0.08	0.39	0.96	2.2	3.1	4.—	6.4

In the 2nd and 3rd experiments every plant was supplied with about

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[4]) J. S. BURD. Rate of absorption of soil-constituents at successive stages of plant-growth. Journ. Agric. Research, Vol. 18. 1919. p. 51—72.

[5]) D. R. HOAGLAND. Relation of the concentration and reaction of the nutrient medium to the growth and absorption of the plant. Journ. Agric. Res., Vol. 18. 1920. p. 73—117.

[6]) D. R. HOAGLAND and J. C. MARTIN. A comparison of sand and solution-cultures with soils and media for plant-growth. Soil science, Vol. 16. 1923. p. 367—388.

[7]) F. W. PARKER. Plant growth and the absorption of phosphorus from culture solutions of different phosphate concentrations (Alabama Agric. Exp. Stat.) Soil science, Vol. 24. p. 129—146.

[8]) F. W. PARKER and W. H. PIERRE. The relation between the concentration of mineral elements in a culture medium and the absorption and utilization of those elements by plants. Soil science, Vol. 25. 1928. p. 337—343.

400 cc of nutrient solution. The nitrogen concentrations of the different culture-series were 14, 42, 70, 126, 182, etc. p. p. m. in these experiments. So the quantities of nitrogen available per plant were respectively 5.6, 16.8, 28.—, 50.—, 73.—, etc. mgrs. From the data of table 1 it may be calculated in how far a renewal of the solutions about every 5th day rendered it possible to vary the nitrogen-concentration during 7 weeks within a suitable range. This appeared to be possible with those cultures in which the available quantity of nitrogen per plant was 40 mgrs or more. If the experiment is continued for no more than 6 weeks, a quantity of nitrogen of about 30 mgrs per plant generally appears to be sufficient.

In the 4th experiment, which was continued for five weeks, the influence was studied of different nitrogen-concentrations increasing from 1.5 to 70 p. p. m. In order to guard the nitrate-concentration in these dilute solutions from too great a fall, respectively to prevent exhaustion of the solution, each plant was given the disposal of 28 mgrs of nitrogen in this experiment. In this way any danger of exhaustion was excluded even in the most dilute solution. The cultures with an amount of 70 mgm. N per liter were again supplied with 400 c.c. of nutrient solution per plant. In the remaining cultures, the nitrate-solutions of which were more dilute, a greater quantum of nutrient solution was given per plant, so that in the different experimental series the volume of the nutrient solution per plant was inversely proportional to the nitrate-concentration.

Apart from the nitrate, the nutrient solutions in all experiments had the same composition and the same concentration. 100 mgrs K_2SO_4, 300 mgrs $MgCl_2 . 6$ aq., 100 mgrs KH_2PO_4, 100 mgrs K_2HPO_4, 400 mgrs $CaSO_4 . 2$ aq. and 10 mgrs $FeSO_4 . 7$ aq. per liter were supplied, which quantities were amply sufficient to prevent deficiency, when the solutions were renewed every 4th to 6th day.

3. Of the nitrates receiving consideration as to the study of the influence of the nitrate-ion-concentration on growth, $NaNO_3$ seemed most suitable to us. From their nature it may be objected to all nitrates, that a change of the nitrate-ion-concentration brings about an equivalent alteration in the concentration of the cation. In judging the harvest-results we have to face the difficulty that it cannot be decided how far the results obtained may be attributed to the NO_3-ion-concentration. Sodium not being necessary for the nutrition of the plant, it could be expected that the growth of the plants would be but slightly dependent on the sodium-ion-concentration. In order to make certain on this subject, the experiments were made in such a way, that the influence of the nitrate-ion and that of the sodium-ion with unequal $NaNO_3$-concentration could be judged separately. Table 2 may give an idea of the experimental arrangement. In the columns 3 and 4 of this table I have given the $NaNO_3$-content and the Na_2SO_4-content of the nutrient solutions used in some culture-series with ascending nitrate-concentration. The numbers of the experimental series in the first column were

made to correspond with the values of the nitrogen-concentrations in p.p.m. (2^{nd} column).

In the 2 last columns there are found the concentrations of the $NaNO_3$ and of the Na_2SO_4 of the culture-solutions expressed in terms of chemical equivalents. The table shows that of each N-concentration two sets of plants were carried out, one of which in the first column has been marked *a*. The solutions of these a-series differ from those of their duplicates in their

TABLE 2.

Number of culture series	Nitrogen concentration in p. p. m.	$NaNO_3$ concentration in p. p. m.	$Na_2SO_4 . 10$ aq. concentration in p. p. m.	$NaNO_3$ concentration in milli-aeq. per L.	Na_2SO_4 concentration in milli-aeq. per L.
14	14	85	—	1	—
14a	14	85	322	1	2
42	42	255	—	3	—
42a	42	255	322	3	2
70	70	425	—	5	—
70a	70	425	644	5	4
126	126	765	—	9	—
126a	126	765	644	9	4
etc.	etc.	etc.	etc.	etc.	

having received an extra-dose of Na_2SO_4. They, therefore, contain an equal amount of nitrogen as their duplicates, but more sodium. The quantity of Na_2SO_4 was chosen in such a way, that the Na-concentration of the a-series was equal to the Na-content of the next series with higher N-concentration. For the rest in all solutions the salts are present mentioned on page 137.

Thus the culture-series 14 and 14*a* correspond in the nitrogen-content but differ in the concentration of sodium. 14*a* contains less nitrogen than 42, but as much sodium, etc. Between the experiment-numbers 70 and 126 there exists a difference of 4 milli-aequivalents per L., that is a twice greater difference than between the series 14, 42 and 70. In accordance with this an extra-dose of Na_2SO_4 of 4 milli-aequivalents was given to series 70*a*.

In anticipation of the results to be discussed in section 5, it may already be mentioned that in this way it could be ascertained that with unequal $NaNO_3$-concentration the growth is not or at most slightly influenced by the Na-concentration, so that the influence of the nitrate-ion-concentration on the growth could be ascertained with fairly great accuracy, when $NaNO_3$ was used as the source of nitrogen.

Yet in another respect this experimental arrangement has done good service in judging the results. In the 2nd and 3rd experiments it was found that the growth strongly decreases, when the N-concentration of the nutrient-solutions exceeds that of 180 p.p.m. (figs 2 and 3, curve *a*). As with ascending nitrate-percentage of the nutrient solutions the total salt-concentration likewise increases, the question might obviously be asked whether the increase of the nitrate-concentration itself, or rather the attending increase of the total salt-concentration should be made responsible for this decrease in growth. Therefore we have to bear in mind that the total salt-content of the solutions of the a-series with equal nitrogen-concentration is higher than that of their duplicates, but equal to that of the next series with higher N-concentration.

If in the curves *a* of figs 2 and 3 in which the weights of the *a*-series are represented by the single points, we compare the weights of the cultures 182, 182*a* and 238 with each other, there is no doubt but the decrease in growth observed between the amounts of N of 182 and 238 p.p.m. should not be attributed to the increase of the total salt-content, but only to the increase of the nitrate-concentration.

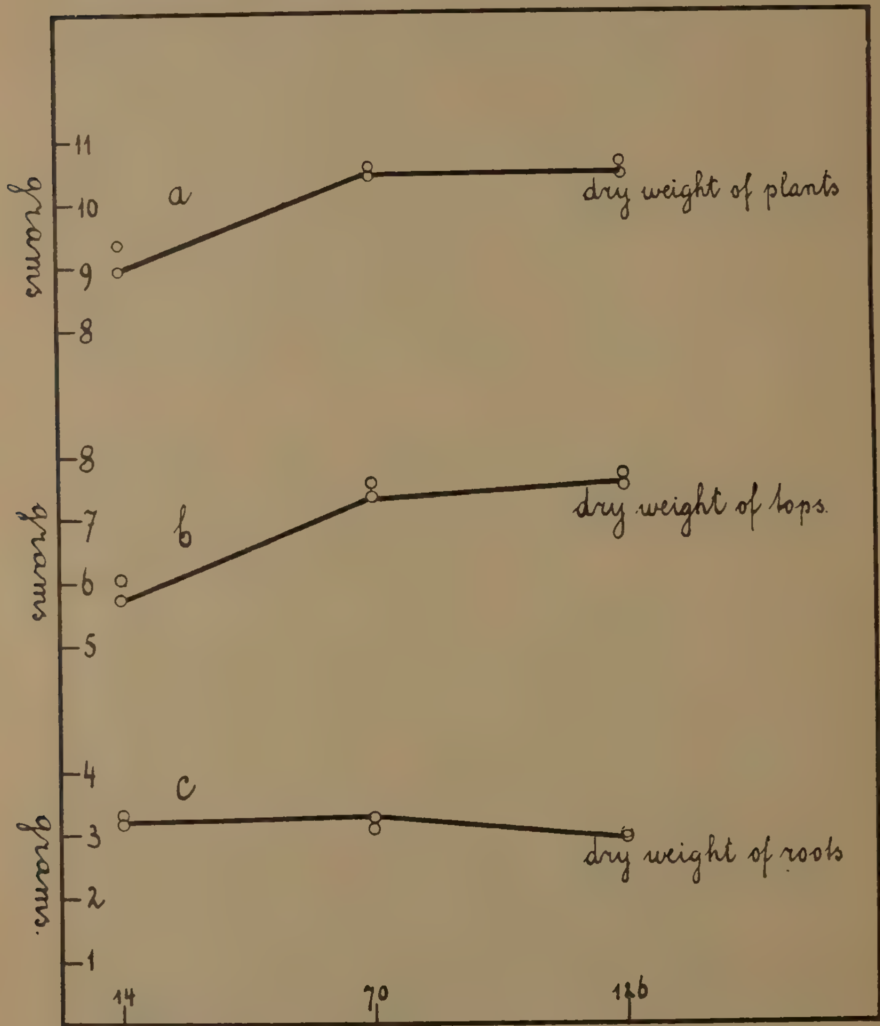
4. The seed used in this research was gathered from a selected pure wheat-strain from SVALÖF ("Sommerkolbenweizen II"). Grains of equal weight were germinated in nutrient solutions; they were sown on hydrophile gauze, which was stretched on floating paraffined wooden rings. After about 6 days seedlings of equal length were transferred to culture-jars, which were put in the greenhouse or out of doors in favourable weather. To the position of the cultures particular attention was given, in order to exclude systematic errors and to reduce accidental errors.

Directly after the harvest the plants of each series were weighed together in a fresh condition. Afterwards when the water-content of the plants did not amount to more than about 5 %, the air-dry-weight was determined of all plants separately, both of the roots and the tops. In connection with the strongly variating moisture-content of air-dry plants, during the drying and the weighing such precautions were taken as to render the air-dry-weights of one and the same experiment comparable. From the weights found of each series the mean error was computed of the average weight of the plants, whilst from the fresh- and dry-weights the percentage dry weight was derived $\left(\frac{\text{dry-weight}}{\text{fresh-weight}} \times 100\right)$. The relative root-weight was likewise determined by expressing the dry-weight of the roots in terms of percentage of the dry-weight of the whole plants. During the experiments the transpiration was determined many times. The hydrogen-ion-concentration of the nutrient-solutions was regularly controlled colorimetrically.

5. The results of the experiments have been graphically represented. The points of the weight- and transpiration-curves give the total weight,

respectively the total transpiration of the culture-series. The single points, not connected by lines, refer to the so-called *a*-series, which were supplied with an extra-dose of sodium in the form of sodium sulphate.

1st *experiment*. The seed was sown as described above on June 5. The



P. p. m. N of nutrient solution

Fig 1.

plants were harvested when they were 31 days old. Only 3 sodium-nitrate-concentrations were compared, viz. 1, 5 and 9 milli-aequivalents per liter. Moreover 3 *a*-series were carried out, the meaning of which has been further explained in section 3. Each experimental series consisted of 14 plants.

The plants throve well. The habitus and the colour of the cultures left nothing to be desired, except the experiment-numbers 14 and 14*a*, which lagged behind in growth after about 3 weeks and adopted the typical yellow colour characteristic of lack of nitrogen. About 340 c.c. of nutrient solution was available per plant. In the 3 series mentioned and their *a*-series, therefore, each plant had respectively 4.7, 23.5, and 42.3 mgrs of nitrogen at its disposal. Since wheat-plants cultivated in nutrient solutions absorb about 1 mgr. of N. per 24 hours in the 3rd week of their growth (see table 1, p. 136), it is evident that the cultures 14 and 14*a* had absorbed about all the nitrate in the course of this week and began to show symptoms of lack of nitrogen.

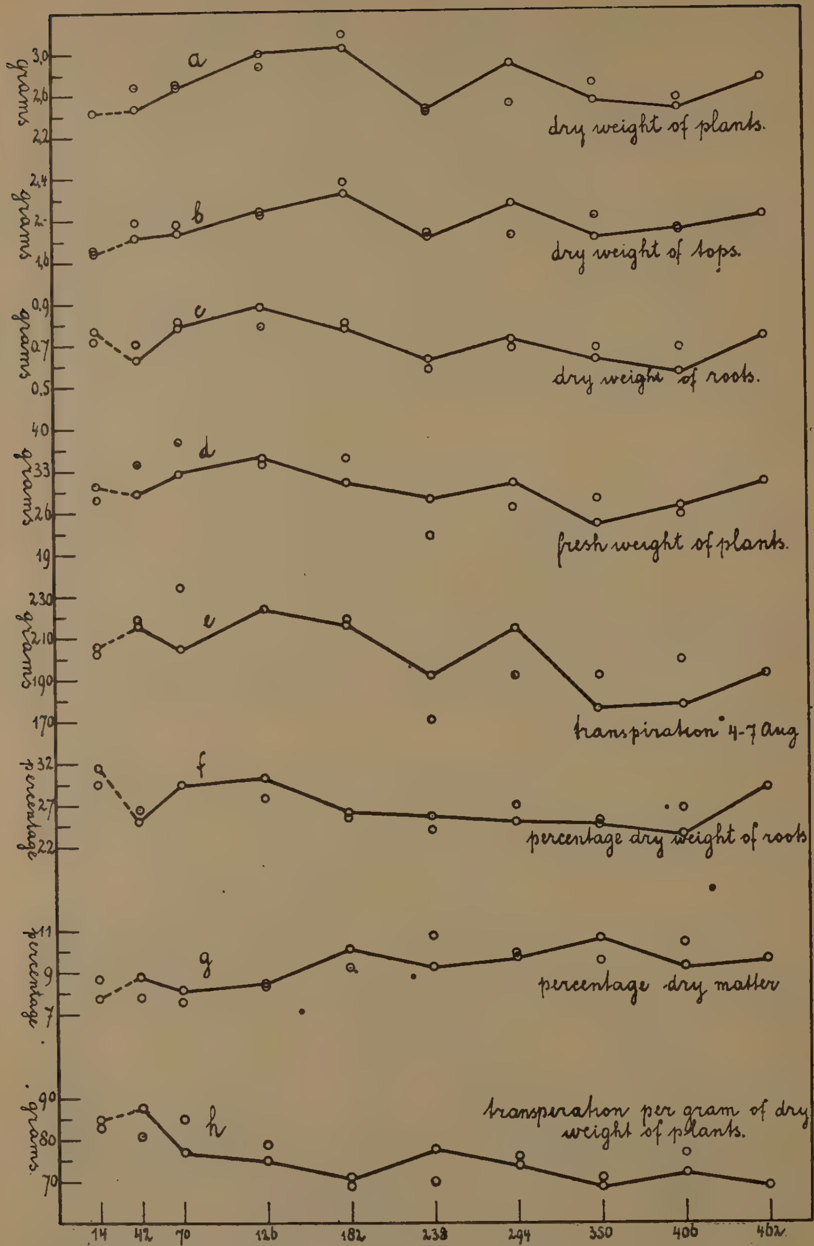
The *results* of this experiment which may appear from the curves of fig. 1 may be summarized as follows : On being harvested the plants cultivated in solutions with a nitrate-nitrogen-concentration of 70 p. p. m. yielded the same dry-weight as the plants the solutions of which had contained an amount of N. of 126 p. p. m. The weights of the *a*-series appeared to correspond with those of their duplicates. From this it may be concluded that the growth of the wheat-plants at least within a certain range of the $NaNO_3$-concentration (in our case 5 and 9 milli-aequivalents per L.) is independent both of the N-concentration and of the Na-concentration of the nutrient solution.

As during the experiment the nitrate-concentration was fluctuating and in the N-solutions of 70 p.p.m. in the last week before harvesting it must have decreased to about 30 p.p.m. and the plants have not responded to this by diminishing their growth, it may be supposed that N-concentrations considerably lower than 70 p.p.m. render a normal growth possible.

Though the weights of the plants in the nutrient solutions with an amount of N. of 14 p.p.m. were found to be smaller than in the other cultures owing to lack of nitrogen, they did agree with the latter in root-weight. Apparently exhaustion of the nutrient solution brings about a relative increase of growth of the root-system, which has indeed been observed before by various investigators.

2nd *experiment*. This was started on July 17. The plants were 22 days old when they were harvested. The range of N-concentration examined extended from 14 to 462 p.p.m. Beside the ordinary series of culture again the *a*-series were carried out. Each experimental series consisted of 12 plants and each of these plants was supplied with 400 c.c. of solution.

Towards the end of the experiment the plants in the solutions with N-concentration of 14 p.p.m. began to show signs of lack of nitrogen. The other cultures seemed fairly normal, and differed to the eye but slightly.



Nitrate nitrogen concentration in parts per million of nutrient solution.

Fig 2.

We only noticed white stripes on the leaves on either side of the midrib in a number of plants in nearly all the series of culture, the cause of which could not be explained.

Results. The graphs *a* to *d* in fig. 2 show that the weights of the plants of the different series of experiments were rather varying. A maximum-growth could be stated in N-concentrations of 182 and 126 p.p.m. From this latter point there appeared to be a regular decrease in growth in the direction of the lower concentrations. There was likewise a difference in growth between the cultures 182 and 238; in the range of the higher N-concentrations (higher than 238 p.p.m.) the growth showed two tops, viz. at 294 and at 462 p.p.m.; maximum growth however was not attained in this range.

On the whole the results of the *a*-series corresponded very well with those of their duplicates; they were now higher now lower than these latter and but for a few exceptions the differences were slight enough compared with the mean error for us to accept that between the *a*-series and their duplicates there was no difference in growth nor in other respects. From this it may be concluded that the variations in growth found between the different series are exclusively due to the nitrate-content of the nutrient solutions and are not influenced by the sodium-concentration.

On our comparing the transpiration-curve (*e* fig. 2) with the growth-curves (*a*—*d*), they appear to correspond to such a degree, that the transpiration at least in the range of concentration studied here, seems to be proportional both to the fresh weights and to the dry weights of the plants.

As far as this result concerns the fresh-weights, it is in accordance with the parallelism between transpiration and fresh weight of the plants ascertained by LIVINGSTON [9]).

With respect to the dry weights HOAGLAND [5]) suggests that the transpiration does not keep pace with it, since this increases per unit of dry weight with decreasing concentration of the nutrient solution. In this experiment which covered but a moderate range of concentration (total concentration 1097 to 4126 p.p.m.) there was at most a very slight decrease of transpiration per gram of dry weight with increasing nitrate-concentration of the solution (*h*, fig. 2). Only in the two weakest solutions the transpiration per unit of dry weight was distinctly greater than in the other cultures.

The course of the curve concerning the percentage dry-weight of the plants (*g*, fig. 2) is reverse to the course of curve *h*. This may be understood when we bear in mind, that LIVINGSTON [9]) found that the transpiration was proportional to the fresh-weight of the plants. We can but expect that variations in percentage dry-weight of the culture-series must be associated with reverse variations in the transpiration per unit of dry weight. From this it follows that strictly speaking transpiration is an unsuitable standard

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[9]) B. E. LIVINGSTON. Relation of transpiration to growth in wheat. Bot. Gaz., Vol. 40 1905. p. 178—195.

[5]) D. R. HOAGLAND, l.c. p. 136.

for judging the relative growth in the sense of relative dry-weight. The variations in percentage dry-weight however appeared to be so slight in the greater part of our cultures that transpiration could serve for judging differences of growth in the range of concentration studied in this experiment. We have determined the transpiration in our experiments from week to week, in order to form an idea of the relative growth of the cultures during vegetation. The first determination was made in the second week of growth. The transpiration-curves not represented here, all showed a similar picture. Thus the variations in growth determined at harvest-time must have appeared shortly after germination.

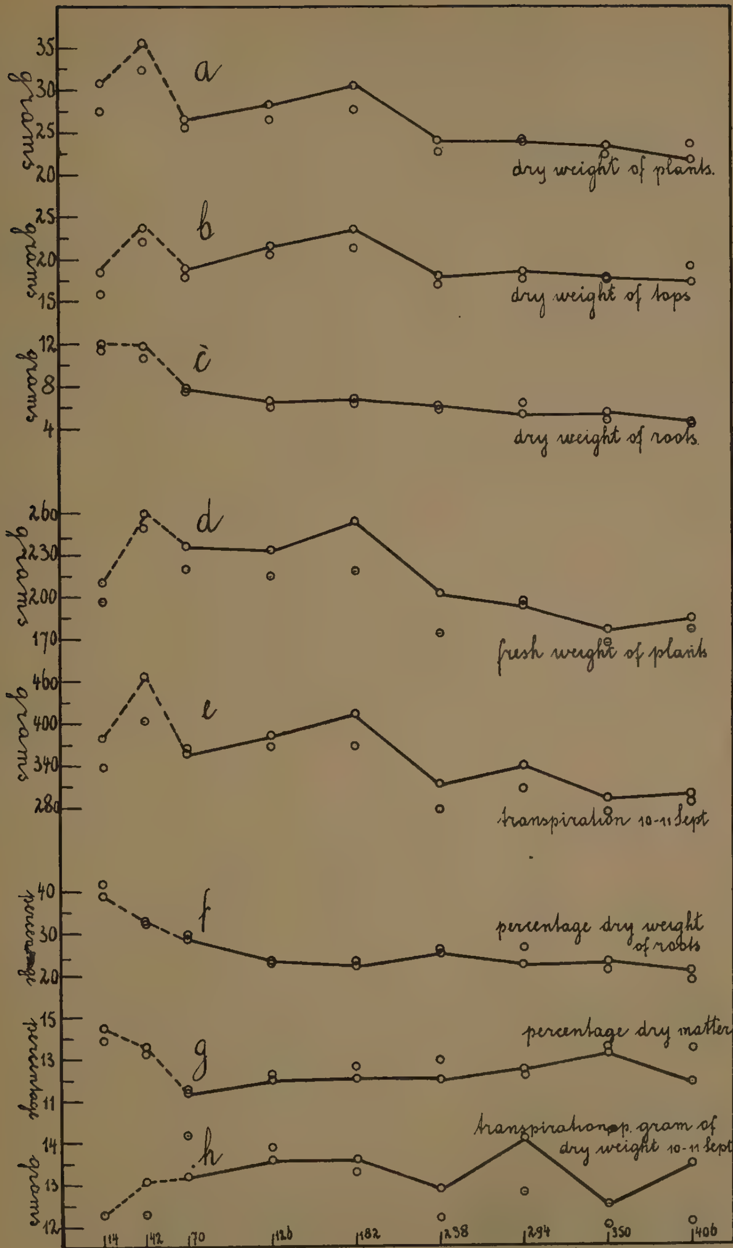
Further it should be observed that in the main the dry-weights of the roots of the different series are directly proportional to those of the whole plants (*a* and *c*, fig. 2). The root-weight expressed in terms of percentage of the total weight of the plant, appeared to diminish but slightly with increasing nitrate concentration of the nutrient solution (*f*, fig. 2).

3rd *experiment.* This experiment started on Aug. 4 was continued for 52 days. The lowest and highest N-concentrations in this experiment were respectively 14 and 406 p.p.m. The dose of nutrient solution per plant was 400 c.c. Each culture-series comprised twelve plants. In this experiment we also carried out *a*-series. The plants developed well, and were apparently but slightly different. At the end of the vegetation however it struck us, that the plants in the higher N-concentrations (238 p.p.m. and more) were smaller than the rest and moreover contrasted unfavourably in habitus.

The plants cultivated in a solution of a N-content of 14 and 42 p.p.m., grew chlorotic already after some weeks on account of lack of nitrogen. We have therefore supplied these cultures, but for some interruptions, more frequently with a new solution than the others, in order to keep the nitrate-concentration up to the mark. From this moment the pH in this series of culture naturally did not rise to the level of the other cultures. On account of this more iron was available, and consequently the plants grew faster. The weights, etc. determined in these cultures are, therefore, unsuitable to be compared with values of the other series of culture.

Results. If we leave the cultures 14 and 42 out of account, it appears that the growth-curves relating to this experiment (fig. 3) correspond in principle with those of the previous experiment. Again there was found a maximum at an amount of N. of 182 p.p.m. and likewise a slight decrease in growth in the direction of the lower N-concentrations and a stronger decrease in growth towards the other side. Again in the range of the higher nitrate-concentrations to begin with an amount of N. of 238 p.p.m., a slighter growth was ascertained than between 70 and 182 p.p.m.

In both experiments the comparatively great difference in growth in the N-solutions of 182 and 238 p.p.m. is noteworthy. This difference was in the two experiments respectively 7.5 and 5.5 $\times$ greater than the mean error of the differences. The values of the *a*-series, though in this experiment on the whole slighter than those of their duplicates deviated so little of these

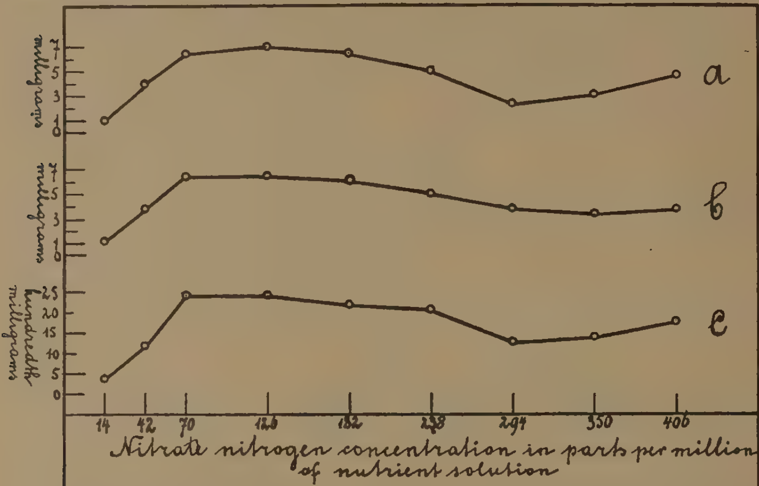


Nitrate nitrogen concentration in parts p. million of nutrient solution.

Fig 3.

latter, that it may be concluded that the growth it not influenced by the concentration of the sodium-ions. The curves *e* to *h* of fig. 3 show about the same picture as the corresponding curves of fig. 2.

The curves represented in fig. 4 refer to the nitrogen-absorption of the plants during the last 5 days of the experiment. Curve *a* gives the average



a - Average daily absorption of nitrogen from nutrient solution 18-24 Sept.
b. Average daily absorption of nitrogen from solution of duplicate series 18-24 Sep.
e. Absorption of nitrogen per gram dry weight of plants.

Fig. 4

nitrogen-absorption of the plants of the different series of culture per 24 hours, curve *b* that of the duplicate-series. The points of this latter curve represent the values of the *a*-series. There is such an accord between these two curves that the nitrogen-absorption may also be considered independent of the sodium-ion-concentration of the nutrient solution. From the values found it may be computed that the solutions of which the initial nitrogen-concentration was about 70 p.p.m., at the end of the last interval of renewal contained hardly any nitrate. As however from these culture-solutions there was absorbed as much nitrogen as from the solutions of the next cultures with higher nitrate-content, they have apparently not suffered any harm from this exhaustion.

The cultures 14 and 42 received no extra renewal of the solution in the experiment during the last 5 days. It may be understood that in these cultures a considerably slighter N-absorption was found, because the solutions of these experimental series must have lost nearly all their nitrate shortly after the renewal.

From curve *c* which gives the nitrogen-absorption per unit of dry-weight it may be concluded that the cultures in the higher nitrate-concentrations show a retardation in this respect as compared with the others. In the main this curve runs parallel with the curve of the dry weights of fig. 3, so that we may obviously expect that the decrease in growth which we stated in the more concentrated nitrate-solutions, was brought about by a lessened nitrogen-absorption.

4th *experiment*. The experiment was started on Aug. 30 and ended on Oct. 3. 8 series of culture were carried out, of which the nitrate-concentration of the nutrient solutions increased from an amount of N. of 1.5 to about 70 p.p.m. with increasing concentration-intervals. As to the experimental arrangement we refer to section 2. The number of plants per experimental series amounted to 12. In the most dilute N-solutions (of about 3 and 5 p.p.m.) however 5 plants were cultivated per series, whilst in the nutrient solution of a N-concentration of 1.5 p.p.m. only one plant was cultivated.

The fluctuations of the nitrate-concentration were determined in all solutions containing an amount of N. of 13 or more p.p.m. The decreases in concentration found in the two last periods of renewal have been summarized in the subjoined table:

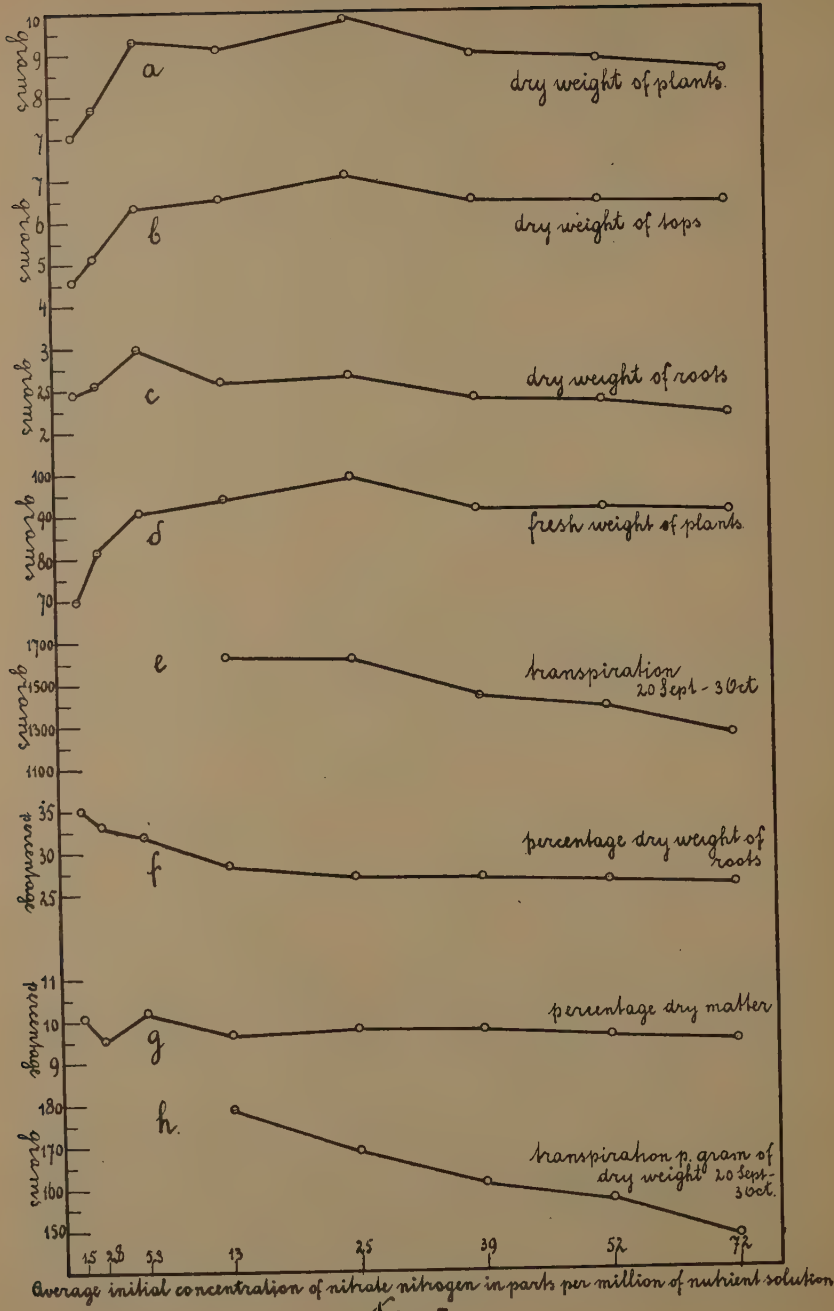
TABLE 3.

Average initial concentration of nitrogen of the growth intervals in p. p. m.	Decrease of nitrogen-concentration in percents of the initial concentration	
	21 — 27 September	27 Sept. — 3 Oct.
13	53 %	69 %
25	48 „	68 „
39	—	55 „
52	42 „	53 „
72	35 „	47 „

No *a*-series were carried out, since in the preceding experiments it had sufficiently appeared that the effect of the $NaNO_3$-concentration on the growth and the nitrogen-absorption of the plants could be exclusively attributed to the nitrate-ion-concentration.

With regard to the pH a deviation was found from the other experiments. Whilst hitherto this exponent during the renewal-intervals increased in all culture-series to an equal degree, this was not the case in this experiment. So it appeared that the pH in the most concentrated solutions had increased from 6.5 to 6.8 during the last interval of growth, whilst that of het most dilute solution had remained stationary.

The explanation of this must be found in the fact, that the plants of the



grams
10
9
8
7
a
dry weight of plants.
grams
7
6
5
4
b
dry weight of tops
grams
3
2.5
2
c
dry weight of roots
grams
100
90
80
70
d
fresh weight of plants.
grams
1700
1500
1300
1100
e
transpiration 20 Sept - 3 Oct
percentage
35
30
25
f
percentage dry weight of roots
percentage
11
10
9
g
percentage dry matter
grams
180
170
160
150
h.
transpiration p. gram of dry weight 20 Sept - 3 Oct.
1.5
2.8
5.3
13
25
39
52
72
Average initial concentration of nitrate nitrogen in parts per million of nutrient solution

Fig. 5.

different culture-series were cultivated in unequal quantities of nutrient solution (SHIVE and BARNETTE [10]).

Results. From the curves of fig. 5 there appears a fair accordance between the weights of the different culture-series. Only the plants cultivated in the most dilute N-solutions (1.5 and 2.8 p.p.m.) were much lighter. During the experiment we had already been struck by the fact that the rate of growth of these plants was considerably retarded as compared with the others, though for the rest they impressed us as being normal. The best growth was found at an amount of N. of 25 p.p.m. From this point the weights decreased slowly and gradually in the direction of the higher concentrations. This decrease is presumably connected with the unequal rise of the pH in the different series of culture. Perhaps the plants in the more dilute solutions have been more successful in obtaining iron. So we have every reason to suppose that we should not have found any maximum of N. at 25 p.p.m., if in all cultures the plants had been supplied with an equal volume of nutrient solution. This is the more likely, as the plants in the more dilute solutions from the nature of things had a greater quantity of oxygen at their disposal than the plants which were cultivated in solutions with greater nitrate-content, i.e. in a smaller volume of nutrient solution.

With regard transpiration (*e* and *h* fig. 5) the behaviour of the plants in this experiment differed from the previous experiments. The transpiration of the plants appeared not to be proportional to their fresh-weight, but to increase both per unit of fresh-weight and per unit of dry-weight with decreasing nitrate-concentration of the nutrient solution. For these low nitrate-concentrations, therefore, the relation found by LIVINGSTON [9]) between transpiration and fresh-weight of the plants does not hold good.

Just as in the previous experiments the relative weight of the roots increased very slowly with decreasing concentration of the nutrient solution. In the weakest solutions, the nitrate-concentration of which was not or hardly sufficient to yield good plants, the relative root-weight seemed to increase on decrease of the nitrate-content at a greater rate. Just as in the previous experiments it appeared in this case that the plant responds to a deficiency in the nutrient medium by relative increase in growth of its root-system.

6. Surveying the results obtained in these experiments by connecting the growth-curves of fig. 5 with the corresponding curves of fig. 1, 2 and 3 we seem permitted to accept that good growth of wheat-plants is possible in nutrient solutions with strongly divergent nitrate-content. The maximum N-concentration for good growth was found by us between 182 and 238 p.p.m., whilst the lowest concentration which permits of rather good growth, appeared to lie below 5 p.p.m. In order to determine this minimum concen-

[10]) R. M. BARNETTE and J. W. SHIVE. The influence of solution volume upon plant growth in relation to reaction change and iron availability in culture solutions. Soil Science, Vol. **15**. 1923. p. 413—425.

[9]) B. E. LIVINGSTON. l.c. p. 143.

tration more accurately, we must account for the decrease experienced by the nitrate-concentration of the solutions with an amount of N. of 5 p.p.m. during the intervals of growth. This decrease we have not determined; from table 3 on page 147 it may however be inferred that the N-content of these solutions has decreased during two last intervals of growth at least to 2.5 p.p.m., i.e. to a value below the initial concentration of the adjacent cultures with lower N-content. As with the plants cultivated in solutions with an initial N-concentration of about 5 p.p.m. rather good growth was stated, it may be approximately accepted, that the minimum N-concentration for satisfactory growth lies below 2.5 and probably above 1.5 p.p.m. The nitrate-nitrogen-concentrations which render good, or at least satisfactory growth possible cover a broad range, the limits of which were found at an amount of N. of about 2 and about 200 p.p.m. of nutrient solution in the experimental conditions described in this research.

Govern. Agric. Experiment Station.

Groningen, January 1929.

Histology. — *Nerve-endings in the muscles of the arm of Sepia officinalis.* By H. BERKELBACH VAN DER SPRENKEL. From the Department of Embryology & Histology of the State-University of Utrecht (Holland). (Director Professor J. BOEKE. M.D.). (Communicated by Prof. J. BOEKE.)

(Communicated at the meeting of January 26, 1929).

Whereas during the last ten years the nerve-endings in vertebrate muscles have been the subject of rather intense investigation, it is surprising that such little work has been done in the field of invertebrates, although it does not lie quite fallow. Most probable the reason for this is that the (medical) histologist is, consciously or unconsciously, inclined to remain as near as possible to the form around which the whole of his scientific effort is constructed, viz.: Homo.

But from a wider scientific point of view, it is most desirable, and perhaps probably necessary in order to gain a clear insight, to make the group of the invertebrates, which contains such a great number of forms, object of modern histological investigation.

A few investigations on nerve-endings in the muscles of invertebrates have certainly been published, but the majority of them is based either on the older gold method (APATHY [1])) or on the methylen-blue technique [2]) so that it appears that the advice of GUÉRIN [3]) (p. 122): il est nécessaire d'étudier les connexions du système nerveux des Céphalopodes dans les matériaux impregnés par les sels d'argent" has not been followed by anybody.

For that reason it was decided to undertake an investigation into the course and endings of nerves in the arm-musculature of Cephalopodes using the BIELSCHOWSKY technique modified by GROS.

The material used was a piece cut transversally from one of the eight ordinary arms of sepia officinalis, which had been preserved in neutral formalin for a number of years. Frozen sections (15—20 μ) are kept in aq. dest. and are passed straight on to 20 % $AgNO_3$. The sections are manipulated with a glass rod around which they can lie easily without overlapping or wrinkling, for wrinkling invariably leads to irregular

1) APATHY, S. Das leitende Element des Nervensystems und seine topographischen Beziehungen zu den Zellen. Mitt. Zool. Station Neapel., Bnd. 12. 1897. p. 495—750.

2) MICHAILOFF. Système nerv. périph. d. Céphalopodes. Bull. Instit. océanographique Monaco. N⁰. 402. 25 Oct, 1921.

3) GUÉRIN. Syst. nerv. et musc. d. l'appareil tenticulaire d. Céphalopodes. Arch. Zool. exp. et générale, IVe Série. T. VIII. p. 1—178. 1908.

impregnation which must be avoided. If however wrinkling does occur, the section should be discarded at once to avoid unnecessary waste of time.

Although the time prescribed for the first impregnation in Agno3 is from 5—30 min., it was found for the material used in this investigation a longer stay for one to two hours gave the best result. From this silver bath the sections are transferred into a dish containing at least 75 cc. of 20 % formalin made by diluting formalin by tap-water. The section is then passed through 2 further similar formalin baths. Itwas observed that if the sections remained longer than 2 à 4 minutes in contact with the formalin a brown rather than a black impregnation was obtained.

With regard to the other steps in the technique, LAWRENTJEW [1]) has already pointed out (p. 469) that the length of the stay in the ammoniacal silver is of the greatest importance in order to get the right impregnation. After a long stay in the 20% silver-nitrate, one must be able to interrupt the action of the NH_3-silvernitrate abruptly, for it may be then a question of a second longer whether the section will be an excellent one or will be spoiled. The best way to do, I think, is to collect sections treated according to different long stays in the different solutions, for then it is possible to have the advantages of all shades of impregnation side to side and one is able to construct an image of reality out of different aspects.

After the impregnation the sections were toned in the usual way, counterstained with haematoxylin and mounted in laevulose-gelatine according to the prescription of HERINGA (Ned. Tijdschr. Geneesk. 1923, 2nd half. p. 448).

From the mass of nervous tissue forming the axis of the arm, nerves appear following the stripes of tissue which subdivide the longitudinal muscle-fibers (cross-sectioned) (conf. figg. 30 and 33 Pl. IV. GUÉRIN). Such a heavy nerve is drawn in fig. 3 (*z*). Looking for the endings of these nerve-threads in the muscle-mass, one finds images as reproduced in the accompanying figures. Fiber-bundles (imbedded in protoplasm) give off delicate branches which often run over great distances amongst the muscle-fibers.

In figure 1 two heavy bundles cross a number of muscle-cells, the protoplasm of the left nerve is distinctly impregnated. Between the 2 bundles runs a very delicate branch which ends in a point situated on (or within) a muscle-cell. Apart from that lies a group of fibrils which shows at least 4 endings : the right and left upper-ones are point-shaped, close to it is a well-formed endring which, except for its minuteness and proportions, is not to be discerned from the "endöse" of the plain muscle-cell in Vertebrates (BOEKE [2]), LAWRENTJEW [3])); the right branch however curls around the muscle-fiber and ends in an endring underneath the muscle-cell. So we learn that one nerve-fiber is able to form as well ring-shaped as point-shaped endrings which must prevent us from estimating the difference in form too high in importance. How these endings lie in relation to the muscle-cell, whether within or without, is indiscriminable here.

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[1]) LAWRENTJEW. Verbreitung d. nerv. Elemente in d. glatten Muskulatur. Zeitschr. mikr.-anat. Forsch., Bnd. 6. '26. p. 467.

[2]) BOEKE, J. Samenhang tusschen zenuweindiging en gladde spiercel. Versl. Kon. Akad. v. Wet. A'dam, 18 Jan. 1915; Noch einmal d. peritermin. Netzwerk u.s.w. Zeitschr. mikr.-anat. Forsch., Bnd. 7. 1926. p. 95. fig. 10.

[3]) LAWRENTJEW. l.c. fig. 14.

Another image of endings is presented to us by figure 2, where 2 heavy bundles run longitudinally over muscle-cells breaking up in a number of thin fibers, many of which have endings, often crowded together. As the right half of the drawing shows us, a delicate fiber may leave a heavy bundle, ending after an elaborate course against a remote muscle-fiber. A remarkable ending is formed by the thread at the left side, an ending which has the shape of a network extending half-cylindrically along the muscle-fiber. Even the highest magnification did not allow me to state the existence of a periterminal network on this spot, although I believe, considering other images, that it is doubtlessly present. The same as is stated by LAWRENTJEW (l.c. p. 478) viz. that ultra-terminal fibers are present, I could plainly see at numerous places, where from an ending a fiber branches off ending on another muscle-fiber. In this drawing impregnation of the protoplasm, in which the nerve-fibrils run as distinct black lines, is very clear.

In addition to the form of the ending the main points of interest are *a.* its situation in relation to surrounding cells and *b.* its physiological task. In other words *a.*: Is the ending intra- or extracellular; *b.*: Is the ending motor or sensory?

Commencing with the latter point, I cannot state with surety of what character these endings are, but where I have found always the same type, I think it quite reasonable (and especially quite simple) to see in them motor endings, the more so as sensory endings in non-striated muscle-fibers are hardly known with certainty. The motor function of an ending is after my opinion only guaranteed by images as given by BOEKE [1]) where the periterminal network continually extends from the ending to the myofibrils. On what base MICHAILOFF calls his ending in l.c. fig. 6 sensory, remains unclear to me, we'll have to wait for his description of a *motor* ending (as he promises us). Upto the moment convincing results compel us to change our opinion, the most cautious thing to do seems to interpret the above described endings as motor.

The question of the intra- or extracellular position of the endings is another-one. The best way to solve this point is to investigate cross-sections, but figure 3 demonstrates how longitudinal sections can inform us just as well. At the left side of the heavy bundle *z*, a delicate thread divides up in several twigs. One of these branches forms an ending (*b*) which carries at an ultraterminal thread a distinct ring-shaped ending (*a*) the latter lying so close to the nucleus of the muscle-cell, that it is unimaginable that the ending should be situated outside of the cell and that the myofibrils, groupwise (see below), should run in between nucleus and ending. A situation, as described above, very probably excludes an extracellular position. By careful focussing the nucleus appears to be bent around the nerve-thread, what indicates a narrow relation between ending and nucleus. The other thread continues its original course and also ends close to the

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[1]) BOEKE, l.c. '26. fig. 3. p. 100.

nucleus of a muscle-cell (*c*). The same drawing shows us a heavy thread that, crossing a number of muscle-fibers, gives off very delicate fibril-bundles forming on the muscle-fibers minute endrings.

The cross-sectioned muscle-fibers show a peculiar order of the fibrils which are radially arranged, so that the cross-section of the muscle-cell shows a clear protoplasmic centre in which the rather large muscle nucleus may be cut (figg. 4 & 5) and around which radially directed lines represent the flat groups of fibrils (c.f. some muscle-cells of fig. 4). It is not at all difficult by scrutinizing to find a great number of endings in a cross-sectioned muscle-group, it is however rare that one of these endings can be used to demonstrate intra- or extracellular position. I believe, however, to have drawn in figure 4 a case where distinctly an ending is so situated in a curve of a nucleus of the muscle-cell, that myofibrils lying between nucleus and ending are to be excluded, this ending has doubtless to be interpreted as intra-cellular.

It strikes the investigator how on the cross-section of a muscle a great number of very delicate threads runs in amongst the muscle-fibers (fig. 4); in order to prevent mistakes, I did not assume such a thread as a nerve thread unless it could be followed up to an unquestionable nervestem.

A last point of importance in the innervation of non-crossstriped muscle-cells are the interstitial cells or lemmoblasts. I am not entitled to assume without further considerations that in this non-vertebrate tissue interstitial cells are present, but the branches of the nerve-fibers show here images which agree completely with those LAWRENTJEW (l.c.) [1]) has published.

It is conspicuous how the slightly thicker fiber-bundles are embedded in protoplasm (which sometimes is impregnated just as well), in which nuclei lie scattered. The fibrils lie close to the nuclei and form an intricate network around it, the nuclei are less regularly shaped and darker impregnated than the nuclei of the plain muscle-cells. So it appears appropriate to interpret cell "a" of fig. 5 as a cell comparable to the interstitial cells of the vertebrates : a kind of lemmoblast. Cell "a" of fig. 4 shows us a lemmoblast situated among the cross-sectioned muscle-fibers. The same figure 5 shows us clearly the inclination of the endings to seek the vicinity of the nuclei ; especially the 3 endings lying at the upper part of the figure are situated close to the nuclei of the cells which they innervate.

The results stated here agree in many points of view with those APATHY communicated in his classic article of 1897. Images as he gives in his figures 1 and 2 of Pl. 32 (Pontobdella) I have got essentially the same : endings lying within the muscle-cell (in Sepia one ending was found which was surrounded by myofibrils just as is indicated in APATHY's double-innervated muscle-cell) ; as APATHY states for Hirudo and Pontobdella, in Sepia often was found that over great distances no ending was to be detected, on another spot many crowded together. Where APATHY however believes

[1]) LAWRENTJEW. l.c. '26. p. 474, fig. 6.; conf. LAWRENTJEW. Proc. Kon. Akad. v Wet. A'dam. Vol. 28. p. 977.

Fig. 1. Nerve-endings on (or within?) muscle-fibers. Ringshaped and other endings formed by one thread. Magn. 1400 times. Zeiss 1.5 mm. Apochrom. Leitz, Periplan. 8 ×

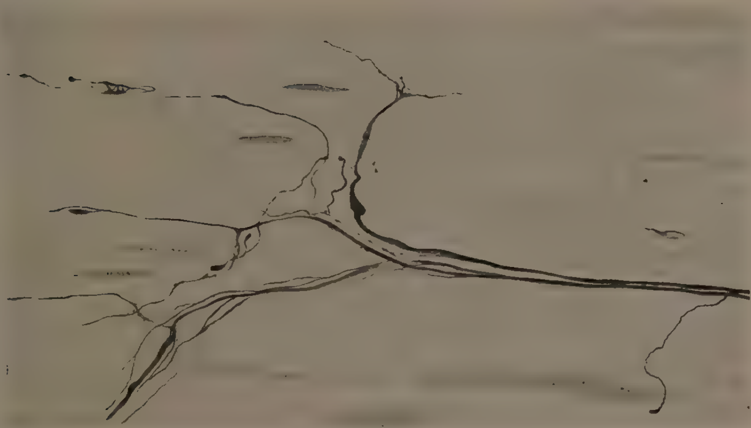

Fig. 2. Nerve-endings on (or within) muscle-cells. At the left side a remarkable form. Magn. 1800 ×. Zeiss. Apochrom. 1.5 mm. Ocul. IV.

Fig. 3. Nerve-endings in muscle-cells, network on muscle-cells. Ring-shaped ending close to the nucleus. Magn. 1800 ×. Zeiss. Apochrom, 1.5 mm. Eyep. IV.

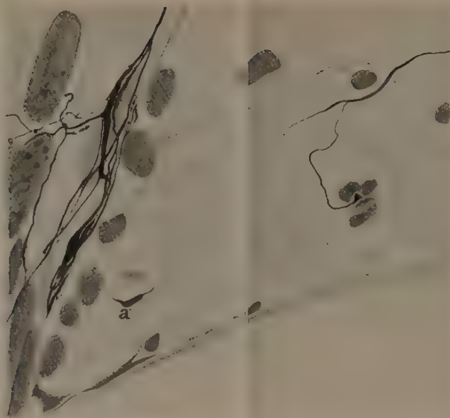

Fig. 4. Ending within a muscle-cell. (cross-sectioned). Nucleus curved. *a*. Interstitial cell. Endings at the left side in the vicinity of the nuclei. Magn. 1375 ×. Leitz. Oil-imm. $^1/_{12}$. Periplan Eyep. Leitz. 8 ×.

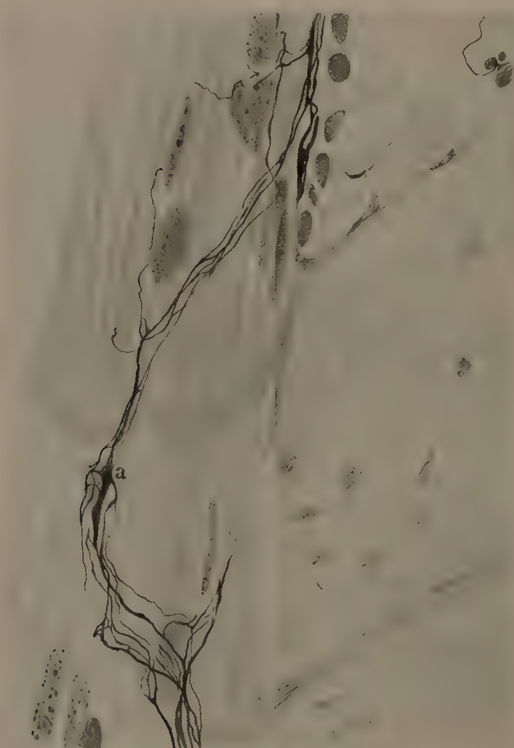

Fig. 5. Nerve on muscle-cells. *a*. Interstitial cell?. Endings in the neighbourhood of the nuclei. Magn. 1000 ×. Leitz. $^1/_{12}$. Peripl. eyepiece. Leitz 8 ×.

(p. 692) that ending and nucleus have no relations whatever to each other, I must disagree with him on this point for there are too many spots where the ending apparently seeks the nucleus than that I can assume the idea that between nucleus ending there would be no more than a casual relation. I, therefore, assume that in Sepia the state of affairs is the same as in Vertebrates where BOEKE (1915, 1926) and LAWRENTJEW have shown that ending and nucleus tend to come in contact as close as possible.

APATHY tells (p. 693) that he has come across many motor threads, which run for a great distance within the muscle-cell in order to leave the cell and to innervate quite another fiber ; although I have given much attention to this point, I have not seen one case of such a relation in the arm of the Cephalopodes.

I believe that this short note is justified as it has been possible to state some points concerning the innervation of the muscle of the arm of the Cephalopodes, viz. :

1°. The nerves end in the muscle-cells in ring-shaped or point-shaped endings.

2°. Very probably the position of the endings is intra-cellular, for preference in the vicinity of the nucleus, which may show a change of form (impressed by the ending).

3°. Cells are present in the course of the fiber-bundles which are comparable to the interstitial cells of the sympathetic innervation system in vertebrates.

Herewith I express my thanks to Professor BOEKE for his continuous interest in every step of this work and for his patience in discussing and criticizing of what I thought to have stated.

Chemistry. — *Investigations about the Structure of Artificial Ultramarines,* IV. *On Ultramarines of Thallium and on the Analogous Derivatives of the Bivalent Metals Calcium, Strontium, Barium, Zinc, Manganese, and Lead.* By Prof. Dr. F. M. JAEGER.

(Communicated at the meeting of December 22, 1928).

§ 1. In this paper the derivatives are described obtained by substitution of the *sodium* in GUIMET's *ultramarine-blue* N°. 7553[1]) by monovalent *thallium*, or by the bivalent metals *calcium, strontium, barium, zinc, manganese* and *lead*. It appeared that the substitution of the *sodium* by the bivalent metals mentioned took place in an *extremely easy* and *complete* way. In some cases we started also with the *silver-ultramarine* corresponding to GUIMET's blue; the products obtained appeared in this case, however, also to contain some *silver*.

The *thallium-ultramarine* was prepared by heating 7 grammes of GUIMET's blue of the formula: $Na_6Al_4Si_6O_{24}S_3$ with 44 gr. *tallous nitrate* and 10 ccm. *water* in sealed CARIUS-tubes for 50 hours at 160° C. The splendidly reddish-violet product thus obtained was carefully washed and purified, and its ROENTGEN-spectrogram was made in the usual way. In another experiment Tl_2SO_4 was used; the product obtained possessed the same colour as the previous one, and its ROENTGEN-spectrogram appeared to be identical with that of the first mentioned preparation. The data obtained are collected in the following table I. (See p. 157.) By the substitution of Na by Tl, the grating-constant a_0 is evidently only slightly diminished; the spectrogram has, for the rest, completely preserved the character of the ordinary *ultramarines*.

§ 2. The *Ca-*, *Sr-*, *Ba-*, *Zn-* and *Mn-ultramarines* were prepared by heating every time 7 Gr. of the *silver-ultramarine*: $Ag_6Al_6Si_6O_{28}S_3$ with five times the theoretical quantity CaI_2, SrI_2, BaI_2, ZnI_2 and MnI_2 and 10 ccm. *water* in sealed tubes at 160° C. for 50 hours. The products obtained were purified in the way described before [2]), by means of KCN, etc. In the case of the *Ca-*, *Sr-* and *Ba*-compounds almost

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[1]) F. M. JAEGER and F. A. VAN MELLE, Proceed. Acad. Sciences Amsterdam, **30**, (1927), 479.

[2]) Loco cit., 483.

pure, olive-green *Ca-*, *Sr-* and *Ba-Ag-ultramarines* of the formula: $Me^{II}Ag_4Al_6Si_6O_{28}S_3$ were formed. On melting these *Ca-* and *Ba*-containing products with CaI_2, respectively BaI_2, *colourless Ca-* and *Ba-ultramarines* were produced, which either contained *no Ag* (*Ca*-compound) at all, or only very little of it (*Ba*-compound). The corresponding *Zn-*

TABLE I

THALLIUM-ULTRAMARINE.

N⁰. of the image:	$2l$ in mm.:	Estim. Intens.:	λ:	$\frac{\theta}{2}$	$\sin^2\frac{\theta}{2}$ (observed)	$\sin^2\frac{\theta}{2}$ (calculated)	Indices:
1	37.2	6	α	12° 0′	0.0432	0.0429	(211)
2	43.6	2	β	14° 4′	0.0591	0.0596	(310)
3	48.6	8	α	15°41′	0.0730	0.0715	(310)
4	52.8	2	α	17° 2′	0.0858	0.0858	(222)
5	57.0	1	α	18°23′	0.0996	0.1001	(321)
6	65.4	7	α	21° 6′	0.1296	0.1287	(330) and (411)
7	85.2	1	α	27°29′	0.2130	0.2145	(521)
8	88.7	1	α	28°37′	0.2294	0.2288	(440)
9	91.6	2	α	29°33′	0.2432	0.2431	(433) and (530)
10	94.5	2	α	30°29′	0.2573	0.2574	(600)
11	97.3	1	α	31°23′	0.2712	0.2717	(532) and (611)
12	105.8	2	α	34° 8′	0.3149	0.3146	(622)
13	118.8	4	α	38°21′	0.3850	0.3861	(633) and (721)
14	124.2	2	α	40° 4′	0.4143	0.4147	(730)
15	128.0	1	α	41°22′	0.4356	0.4433	(732)
16	146.6	2	α	47°17′	0.5401	0.5434	(662)
17	159.0	1	α	51°23′	0.6092	0.6149	(655) and (761)

Radius of the Camera: 44.4 mm. Voltage: 55 K.V. Time of Exposition: 2 hours.

Quadratic Equation: $\sin^2\frac{\theta}{2} = 0.00715 \cdot (h^2 + k^2 + l^2)$, for the *Cu-α*-radiation.

$$\sin^2\frac{\theta}{2} = 0.00596 \cdot (h^2 + k^2 + l^2),$$ for the *Cu-β*-radiation.

Grating-constant: $a_0 = 8.98$ A.U. $\lambda_\alpha = 1.540$ A.U.; $\lambda_\beta = 1.318$ A.U.

TABLE II.

CALCIUM-SODIUM-SILVER-ULTRAMARINE.

Contains: 31.2 % Ag, and is pure: $CaNa_2Ag_4Al_6Si_6O_{28}S_3$.

N⁰. of the image:	$2l$ in mm.:	Estim. Intens.:	λ:	$\frac{\theta}{2}$:	$sin^2 \frac{\theta}{2}$ (observed)	$sin^2 \frac{\theta}{2}$ (calculated)	Indices:
1	37.5	4	α	12° 6′	0.0439	0.0442	(211)
2	44.3	1	β	14°17′	0.0608	0.0600	(310)
3	49.2	9	α	15°53′	0.0749	0.0737	(310)
4	53.6	3	α	17°18′	0.0884	0.0885	(222)
5	58.7	3	α	18°56′	0.1053	0.1033	(321)
6	66.2	9	α	21°21′	0.1325	0.1327	(330) and (411)
7	69.4	1	α	22°23′	0.1450	0.1470	(420)
8	73.5	1	α	23°43′	0.1618	0.1622	(332)
9	90.2	2	α	29° 6′	0.2365	0.2360	(440)
10	93.2	2	α	30° 4′	0.2510	0.2507	(433) and (530)
11	95.9	1	α	30°56′	0.2642	0.2655	(600)
12	99.0	1	α	31°56′	0.2798	0.2802	(532) and (611)
13	105.0	1	α	33°52′	0.3105	0.3097	(541)
14	107.3	1	α	34°36′	0.3224	0.3245	(622)
15	121.0	3	α	39° 2′	0.3966	0.3982	(633) and (721)
16	126.1	1	α	40°40′	0.4248	0.4277	(730)

Radius of the camera: 44.4 mm. $V = 55000$ Volt. Time of Expositon: 3 hours.

$\lambda_\alpha = 1.540$ A.U.; $\lambda_\beta = 1.318$ A.U. $a_0 = 8.97$ A.U.

Quadratic Equation:

$$sin^2 \frac{\theta}{2} = 0.007375\,(h^2 + k^2 + l^2), \text{ for the } \alpha\text{-radiation.}$$

$$sin^2 \frac{\theta}{2} = 0.005996\,(h^2 + k^2 + l^2), \text{ for the } \beta\text{-radiation.}$$

ultramarine is also practically colourless, with a very feeble, pale violet hue. In the case of MnI_2, a pale, yellowish-brown product was generated, which contained less Al than the original *Ag-ultramarine*; for the greater part it consisted of an *ultramarine*: $Na_2Mn_2Al_4Si_6O_{28}S_3$, to which some *Ag-ultramarine* was admixed. Later on we will return to the presumable

TABLE III.
CALCIUM-ULTRAMARINE.
The white compound does not contain any more Ag.

No. of the image:	$2l$ in mm.:	Estim. Intens.:	λ:	$\frac{\theta}{2}$	$sin^2 \frac{\theta}{2}$ (observed)	$sin^2 \frac{\theta}{2}$ (calculated)	Indices:
1	37.2	6	α	12° 0′	0.0432	0.0435	(211)
2	42.6	9	α	13°46′	0.0565	0.0581	(220)
3	45.0	2	β	14°30′	0.0627	0.0600	(310)
4	48.9	3	α	15°46′	0.0738	0.0726	(310)
5	54.0	2	α	17°25′	0.0896	0.0871	(222)
6	61.5	1	α	19°50′	0.1151	0.1162	(400)
7	65.6	1	α	21°10′	0.1304	0.1307	(330) and (411)
8	69.8	2	α	22°31′	0.1466	0.1452	(420)
9	73.0	1	α	23°33′	0.1596	0.1597	(332)
10	76.0	1	α	24°31′	0.1722	0.1742	(422)
11	79.0	2	α	25°29′	0.1851	0.1888	(431) and (510)
12	86.7	2	α	27°58′	0.2199	0.2178	(521)
13	90.0	2	α	29° 2′	0.2355	0.2323	(440)
14	93.9	2	α	30°17′	0.2543	0.2469	(433) and (530)
15	97.0	1	α	31°17′	0.2696	0.2614	(600)
16	101.5	1	α	32°44′	0.2924	0.2904	(620)
17	107.4	1	α	34°39′	0.3233	0.3194	(622)
18	109.9	1	α	35°27′	0.3364	0.3340	(631)
19	113.9	2	α	36°45′	0.3580	0.3630	(543); (505); (710)
20	119.8	1	α	38°39′	0.3901	0.3920	(633) and (721)
21	126.0	1	α	40°39′	0.4244	0.4211	(730)
22	130.0	1	α	41°56′	0.4466	0.4501	(651) and (732)
23	138.0	1	α	44°31′	0.4916	0.4937	(644) and (820)
24	148.0	1	α	47°45′	0.5479	0.5518	(622)

Spectrogram with broad and hazy lines; inaccurate measurements.
Radius of the camera: 44.4 mm. $\lambda_\alpha = 1.540$ A.U.; $\lambda_\beta = 1.318$ A.U. $V = 55000$ Volt.
Time of Exposition: 3 hours.

Quadratic Equation: $sin^2 \frac{\theta}{2} = 0.00726\,(h^2 + k^2 + l^2)$ for the α-radiation. $a_0 = 9.04$ A.U.

cause of this *sodium*-content. The data relating to the powder-spectrograms obtained with these products, are collected in the tables II—VII; the powder-spectrogram of the *Ba*-compound was feeble and hazy; that of the *Mn*-compound had a black-ground, as a consequence of the *Mn*-radiation excited.

Also these products all showed the original type of the *ultramarines.* Evidently in the case of the *Ca*-, *Sr*- and *Ba*-salts, exactly $^1/_3$ of the *silver* in *silver-ultramarine* is easily replaced by 1 atom of the bivalent

TABLE IV.

STRONTIUM-SODIUM-SILVER--ULTRAMARINE.

Contains: 29 % *Ag* and is: $SrNa_2Ag_4Al_6Si_6O_{28}S_3$ (30 % *Ag*).

N⁰. of the image:	$2l$ in mm.:	Estim. Intens.:	λ:	$\frac{\theta}{2}$:	$\sin^2\frac{\theta}{2}$ (observed)	$\sin^2\frac{\theta}{2}$ (calculated)	Indices:
1	38.5	6	α	12°25′	0.0461	0.0444	(211)
2	44.0	2	β	14°12′	0.0602	0.0601	(310)
3	49.2	7	α	15°52′	0.0747	0.0740	(310)
4	53.5	1	α	17°16′	0.0881	0.0887	(222)
5	58.5	1	α	18°52′	0.1045	0.1036	(321)
6	66.6	6	α	21°28′	0.1339	0.1331	(330) and (411)
7	74.2	1	α	23°56′	0.1646	0.1627	(332)
8	77.8	3	α	25°6′	0.1799	0.1775	(422)
9	88.0	2	α	28°23′	0.2260	0.2219	(521)
10	92.1	2	α	29°43′	0.2457	0.2515	(433) and (530)
11	96.0	1	α	30°58′	0.2648	0.2663	(600)
12	98.0	1	α	31°37′	0.2749	0.2810	(532) and (611)
13	121.7	2	α	39°16′	0.4005	0.3995	(633) and (721)

Radius of the camera: 44.4 mm. $V = 55000$ Volt. Time of Exposition: 3 hours.

$\lambda_\alpha = 1.540$ A.U.; $\lambda_\beta = 1.318$ A.U. $a_0 = 8.95$ A.U.

Quadratic Equation: $\sin^2\frac{\theta}{2} = 0.007397\,(h^2 + k^2 + l^2)$, for the α-radiation.

$\sin^2\frac{\theta}{2} = 0.006013\,(h^2 + k^2 + l^2)$, for the β-radiation.

TABLE V

POWDERSPECTROGRAM OF BARIUM-SODIUM-ULTRAMARINE.

Contains: 1.86 % *Ag*: consists of nearly pure: $Na_2Ba_3Al_6Si_6O_{28}S_3$.

Nº. of the image:	$2l$ in mm.:	Estim. Intensity:	λ	$\Sigma(\overline{h^2})$:	$\frac{\theta}{2}$:	$sin^2 \frac{\theta}{2}$ (observed)	$sin^2 \frac{\theta}{2}$ (calculated)	Indices:
1	37.7	4	α	6	12°10′	0.0444	0.0442	(211)
2	(43.8)	2	β	10	14° 9′	0.0597	0.0598	(310)
3	(48.8)	2	α	10	15°45′	0.0736	0.0736	(310)
4	53.6	4	α	12	17°17′	0.0883	0.0883	(222)
5	58.0	1	α	14	18°43′	0.1030	0.1030	(321)
6	62.3	1	α	16	20° 6′	0.1181	0.1178	(400)
7	66.3	3	α	18	21°23′	0.1329	0.1325	(330) and (411)
8	70.0	2	α	20	22°35′	0.1475	0.1472	(420)
9	73.7	2	α	22	23°44′	0.1620	0.1619	(332)
10	77.1	1	α	24	24°52′	0.1768	0.1766	(422)
11	86.6	2	α	30	27°56′	0.2195	0.2208	(521)
12	90.6	2	α	32	29°15′	0.2385	0.2355	(440)
13	(98.9)	2	α	38	31°55′	0.2795	0.2797	(532) and (611)
14	113.2	2	α	48	36°31′	0.3541	0.3533	(444)
15	(126.6)	1	α	58	40°50′	0.4275	0.4269	(730)

Radius of the camera: 44.4 mm. $V = 55000$ Volt. Time of Exposition: 2 hours.

$\lambda_\alpha = 1.540$ A.U.; $\lambda_\beta = 1.388$ mm. $a_0 = 8{,}97$ A.U.

Quadratic Equation: $sin^2 \frac{\theta}{2} = 0.00736\ (h^2 + k^2 + l^2)$. for the α-radiation.

$$sin^2 \frac{\theta}{2} = 0.00598\ (h^2 + k^2 + l^2), \text{ for the } \beta\text{-radiation.}$$

The film gave only hazy and broad lines; consequently only approximate estimations.

metal; but on melting these products with an excess of the *iodides*, practically *all Ag* is substituted.

The relative intensities of corresponding diffraction-images in these spectrograms are graphically represented in Fig. 1 in the usual way.

With the exception of the *Ca*- and *Sr*-compound, also in these cases no general relations seem to be present. Although the original character of the *ultramarine*-spectrograms is preserved throughout, yet the relative intensities of the images is different from those of the ordinary *ultramarines*.

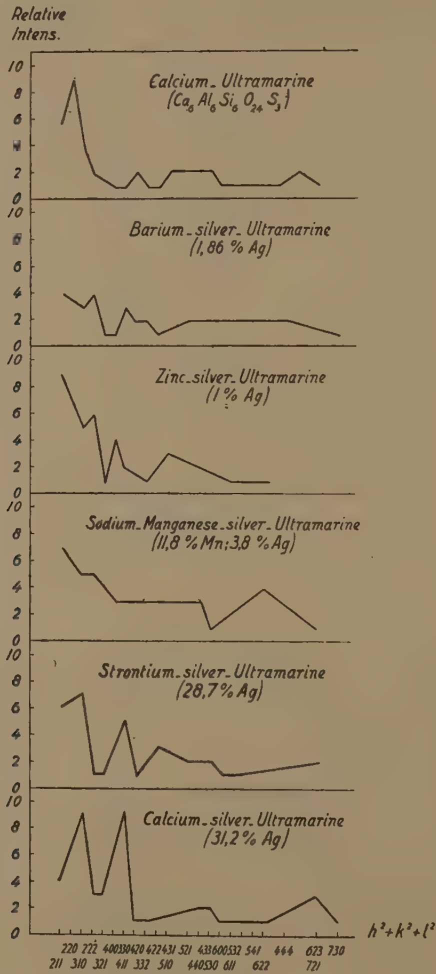

Fig. 1.

§ 3. In the case of the heavy metals, a *lead-sodium-ultramarine* rich in silica was prepared, by heating GUIMET's blue: $Na_6Al_4Si_6O_{24}S_3$ in

sealed tubes at 160° C. for 50 hours with five times the quantity of $Pb(NO_3)_2$, theoretically necessary for a complete substitution of the *sodium* present. After careful purification, a powder-spectrogram of the

TABLE VI.

ZINC-SODIUM-ULTRAMARINE.

(Still contains: 1 % *Ag*. colour: very pale violet, almost withe).

Is almost pure: $Na_2Zn_3Al_6Si_6O_{28}S_3$.

N⁰. of the image:	$2l$ in mm.:	Estim. Intens.:	λ:	$\frac{\theta}{2}$	$sin^2 \frac{\theta}{2}$ (observed)	$sin^2 \frac{\theta}{2}$ (calculated)	Indices:
1	38.0	9	α	12°15′	0.0450	0.0452	(211)
2	49.1	5	α	15°50′	0.0744	0.0753	(310)
3	54.3	6	α	17°30′	0.0904	0.0904	(222)
4	58.9	1	α	19° 1′	0.1062	0.1054	(321)
5	63.4	4	α	20°28′	0.1223	0.1205	(400)
6	66.8	2	α	21°33′	0.1349	0.1355	(330) and (411)
7	74.2	1	α	23°56′	0.1646	0.1657	(332)
8	81.6	3	α	26°19′	0.1965	0.1958	(431) and (510)
9	90.9	2	α	29°19′	0.2398	0.2410	(440)
10	100.1	1	α	32°17′	0.2853	0.2861	(532) and (611)
11	109.0	1	α	35°10′	0.3317	0.3313	(622)

Radius of the camera: 44.4 mm. $V = 55000$ Volt. Time of Exposition: 3 hours.

$\lambda_\alpha = 1.540$ A.U. $a_0 = 8.87$ A.U.

Quadratic Equation: $sin^2 \frac{\theta}{2} = 0.00753\,(h^2 + k^2 + l^2)$, for the α-radiation.

product was thus obtained; the results of its analysis are put together in table VIII.

By melting *Ag-ultramarine* with $PbCl_2$ and extraction of the residual $PbCl_2$ by means of boiling water, a dark greyish *ultramarine*, containing *Pb* and *Ag* simultaneously, could be obtained; its powder-spectrogram showed the data collected in Table IX.

TABLE VII.

MANGANESE-SODIUM-SILVER-ULTRAMARINE.

Contains: 3.8 % *Ag* admixed; 11.3 % *Mn*; 4.3 % *Na*, 10 % *Al*; 17 % *Si* and is almost pure: $Na_2Mn_2Al_4Si_6O_{28}S_3$.

N⁰. of the image:	$2l$ in mm.:	Estim. Intens.:	λ:	$\frac{\theta}{2}$:	$\sin^2 \frac{\theta}{2}$ (observed)	$\sin^2 \frac{\theta}{2}$ (calculated)	Indices:
1	37.4	7	α	12°4′	0.0437	0.0435	(211)
2	48.4	5	α	15°37′	0.0725	0.0725	(310)
3	53.0	5	α	17°6′	0.0865	0.0870	(222)
4	62.0	3	α	20°1′	0.1172	0.1160	(400)
5	65.8	3	α	21°12′	0.1310	0.1305	(411) and (330)
6	72.4	3	α	23°22′	0.1573	0.1595	(332)
7	79.8	3	α	25°45′	0.1887	0.1885	(431) and (510)
8	88.7	3	α	28°37′	0.2294	0.2320	(440)
9	91.9	1	α	29°39′	0.2447	0.2465	(433) and (530)
10	106.3	4	α	34°19′	0.3175	0.3190	(622)
11	119.9	1	α	38°41′	0.3906	0.3915	(633) and (721)

Radius of the camera: 44.4 mm. $V = 55000$ Volt. Time of Exposition: 2 hours.

$\lambda_\alpha = 1.540$ A.U. $a_0 = 9.04$ A.U.

Quadratic Equation: $\sin^2 \frac{\theta}{2} = 0.00725\,(h^2 + k^2 + l^2)$, for the α-radiation.

§ 4. Attempts were made to prepare in an analogous way the *mercurous* and *mercuric-ultramarines* from GUIMET's blue by means of *mercurous-*, respectively *mercuric-nitrate*. At 160° C. in the case of the *mercurous* salt a pale canary-yellow product was obtained; in the case of the *mercuric* salt a dull, pale brownish-yellow compound. These products, however, appeared to be mixed with basic salts of mercury, from which they could not be freed completely, not even after being washed for a long time with a 10 % solution of *acetic acid*, towards which these compounds seem to be stable, at least *at low temperatures*. However, *sulphides* of mercury were *not* generated in these reactions. But it seems, that a more profound decomposition of the *ultramarine* used occurs in

these reactions, the nature of which is not yet clear. The powder-spectrograms obtained appeared to be *different* from the usual ones and seem *not* to correspond to cubic symmetry. We are occupied in studying these reactions, and also those taking place with other bivalent metals, more in details.

TABLE VIII.

YELLOW LEAD-SODIUM-ULTRAMARINE.

(Contains: 43.4 % Pb; calculated for: $Pb_3Na_2Al_4Si_6O_{24}S_3$: 42.9 % Pb.

N⁰. of the image:	$2l$ in mm.:	Estim. Intensit.:	λ:	$\frac{\theta}{2}$:	$\sin^2\frac{\theta}{2}$ (observed)	$\sin^2\frac{\theta}{2}$ (calculated)	Indices:
1	37.5	2	α	12° 6′	0.0438	0.0426	(211)
2	43.1	3	α	13°54′	0.0577	0.0568	(220)
3	48.1	9	α	15°31′	0.0715	0.0710	(310)
4	52.7	7	α	17° 0′	0.0855	0.0852	(222)
5	57.6	1	α	18°35′	0.1015	0.0994	(321)
6	64.7	8	α	20°52′	0.1269	0.1278	(330) and (411)
7	79.5	1	α	25°37′	0.1869	0.1846	(431) and (510)
8	88.0	2	α	28°23′	0.2260	0.2272	(440)
9	91.0	3	α	29°21′	0.2402	0.2414	(530) and (433)
10	93.8	2	α	30°15′	0.2538	0.2556	(600)
11	96.7	3	α	31°12′	0.2684	0.2698	(532) and (611)
12	105.3	1	α	33°58′	0.3122	0.3124	(622)
13	114.0	1	α	36°43′	0.3574	0.3550	(505) (543) and (710)
14	118.3	4	α	38°10′	0.3818	0.3834	(633) and (721)
15	146.3	1	α	47°12′	0.5383	0.5396	(662)

Radius of the camera: 44.4 mm. $V = 50000$ Volt. Time of Exposition: 2 hours.

$\lambda_\alpha = 1.540$ A.U.; $\lambda_\beta = 1.318$ A.U.; $a_0 = 9.14$ A.U.

Quadratic Equation: $\sin^2\frac{\theta}{2} = 0.00710\ (h^2 + k^2 + l^2)$, for the α-radiation.

$\sin^2\frac{\theta}{2} = 0.00577\ (h^2 + k^2 + l^2)$, for the β-radiation.

TABLE IX.

LEAD-SILVER-ULTRAMARINE.

Obtained by melting *Ag-Ultr.* with $PbCl_2$ and extraction with water; still contains *Ag*. Colour: dark greenish-grey.

N^0. of the image:	$2l$ in mm.:	Estim. Intens.:	λ:	$\frac{\theta}{2}$:	$sin^2 \frac{\theta}{2}$ (observed)	$sin^2 \frac{\theta}{2}$ (calculated)	Indices:
1	36.7	5	α	11°50′	0.0421	0.0427	(211)
2	43.2	2	β	13°54′	0.0578	0.0588	(310)
3	47.7	9	α	15°22′	0.0702	0.0712	(310)
4	53.1	7	α	17° 8′	0.0868	0.0854	(222)
5	57.8	7	α	18°39′	0.1023	0.0997	(321)
6	65.0	7	α	20°58′	0.1280	0.1282	(330) and (411)
7	67.8	1	α	21°58′	0.1400	0.1424	(420)
8	72.1	1	α	23°16′	0.1560	0.1566	(332)
9	75.7	1	α	24°25′	0.1709	0.1709	(422)
10	79.6	1	α	25°41′	0.1878	0.1851	(431) and (510)
11	83.1	2	α	26°48′	0.2033	0.2136	(521)
12	88.6	1	α	28°35′	0.2289	0.2278	(440)
13	91.2	2	α	29°25′	0.2412	0.2421	(433) and (530)
14	94.4	3	α	30°27′	0.2568	0.2563	(600)
15	97.3	2	α	31°23′	0.2712	0.2706	(532) and (611)
16	118.5	3	α	38°14′	0.3830	0.3845	(633) and (721)

Radius of the camera: 44.4 mm. Voltage: 55 K.V.. $a_0 = 9.12$ A.U.
Time of Exposition: 2 hours.

Quadratic Equation: $sin^2 \frac{\theta}{2} = 0.00712\,(h^2 + k^2 + l^2)$, for the α-radiation.

$sin^2 \frac{\theta}{2} = 0.00588\,(h^2 + k^2 + l^2)$, for the β-radiation.

Finally it may be remarked here, that the *magnesium-ultramarine* has a pale blue colour; in this respect it is obviously different from the other *ultramarines* derived from the bivalent metals.

Groningen, *Laboratory for Inorganic and Physical Chemistry of the University.*

Chemistry. — *Investigations about the Structure of the Artificial Ultramarines. V. On Absorption-phenomena with Ultramarine and on the Structure of Nosean, Hauyne and the Ultramarines.* By Prof. F. M. JAEGER and F. A. VAN MELLE.

(Communicated at the meeting of December 22, 1928).

§ 1. In a series of previous papers [1]) it was already mentioned that on heating *Ag-ultramarine* with solutions of *alcali-halogenides*, equilibria are reached which are determined by the concentration of the solutions and by the temperature. These equilibria are only slowly established and it is worth while studying the velocity of the substitution of the *Ag* in these compounds by the *alcali*-metals used.

For this purpose in each experiment 7 Gr. of *Ag-ultramarine* were heated in sealed CARIUS-tubes with 24 Gr. *NaI* and 10 Gr. water at 100° C. in a thermostat, and the contents of the tubes studied after 1, 2, 3...50 hours' heating. After the usual purification of the products obtained, the *Ag*-content was determined, and from these numbers the amount of *Ag* replaced was afterwards calculated. The first products, for instance after 1 hour's heating, were coloured pale bluish-green, the subsequent ones pale blue; and it appeared that their colour gradually became darker, as the duration of the heating was more prolonged. (Series *A*).

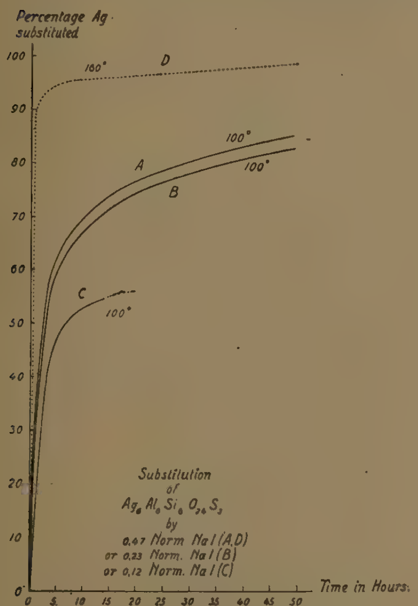

Fig. 1.

Then the same experiments were repeated with solutions possessing only half the concentration of the first (Series *B*). The results are summarized in the following table I and graphically represented in Fig 1.

[1]) F. M. JAEGER, H. G. K. WESTENBRINK and F. A. VAN MELLE. Proc. Acad. Sciences Amsterdam, **30**, (1927), 249; F. M. JAEGER and F. A. VAN MELLE, ibidem, 479; F. M. JAEGER, ibidem, 885.

TABLE I.

Velocity of Reaction if Ag-Ultramarine is treated with Solutions of Sodium-iodide at 100° C.

Duration of Heating in Hours:	*A.* Percentage of the Silver originally present, which has been substituted by Sodium:	*B.* Percentage of the Silver originally present, which has been substituted by Sodium:
1	32%	30%
2	48	46
3	52.5	50.6
4	58	55.7
5	60.7	58.2
6	63	60
7	64.5	62
8	65.7	63.3
9	67	64.5
10	68	65.7
15	72.7	70.3
23	76.6	74.5
36	81	79
50	84.5	81.6

C. If only 2.4 Gr. *NaI* are taken, at 100° C. in 5 hours 46%, in 10 hours 52.5% of the *Ag* present was replaced (concentration about 1/3 of *A*).

D. At 160° C. in the case *A* in 5 hours already 94.9% and after 50 hours 97.7% of the *Ag* appeared to be substituted by *sodium.*

From these data it follows:

1. That the substitution occurs most rapidly during the first period of the reaction and the more rapidly, as the concentration of the solutions used is greater.

2. That the shape of the curves indicates the occurrence of absorption-phenomena, i.e. of processes going on at the surface of the finely powdered *ultramarine.* Evidently a state of saturation of the surface is gradually reached; the substitution takes place the more slowly, as more *Ag*-atoms have already been substituted and occurs the more rapidly, as the concentration of the solutions used is greater or the temperature higher.

The influence of such absorption-phenomena doubtlessly manifests itself

in the irregularities often met with in this kind of experiments: the greater or less degree of dispersion of the solid *Ag-ultramarine* certainly plays an important part in them. Later on we shall return more in detail to the causes of this absorption.

§ 2. An attempt was made in the following way to determine the structure of *nosean* (*hauyne*) and of the *ultramarines*.

In recent times W. L. BRAGG [1]) has determined the structure of a number of *silicates*. It has become clear that this structure is for a good deal determined by a close arrangement of the *oxygen*-ions present, the diameter of their sphere of action being pretty constantly 2,6 or 2,7 A.U. It was natural to try, if the dimensions of the elementary cell as found by us: 9,1 A.U. could be brought into relation with a tetrahedral arrangement of *O*-ions, in the centre of which the *Si*-ion with its much smaller volume then could find its place. Indeed, the distance of the two middles of the opposite edges in a tetrahedron of contiguous *O*-ions, is found to be: 4,55 A.U., — i.e. exactly *half* of the height of the elementary cell of *nosean* and the *ultramarines*, The simplest formula for *nosean* requires that within the elementary cell a mass: $Na_{10}Al_6Si_6O_{32}S_2$ be included; i.e. 32 *O*-atoms (or ions) must find a place in it, which number is exactly 8×4 atoms. However, a really "closest" packing of *O*-ions cannot be present in this case: for the volume of such an *O*-ion would here be: 23,5 A.U.^{3}, while for a tetrahedrally arranged *O*-ion, this volume cannot be greater than 14 A.U.^{3} Moreover, such a closest packing seems improbable with respect to the low refractive index of these *silicates* (about 1,5 for *Na*-light), which in the case of such an arrangement would have a value of about 1,7; and finally also, because of the presence of many *Na*-atoms (or ions), which by no means could find a place within the interstices left in such an arrangement.

It was previously demonstrated [2]) that the spare-groups possible here, are:

$$T^3;\ T^5;\ T_h^5;\ T_d^3;\ O^5 \text{ en } O_h^9,$$

a definite choice between them being impossible, because the true symmetry of these *silicates*, which is probably either hexacis-octahedral or hexacis-tetrahedral, is unknown. It was, however, already pointed out, that in every case no "fixed" position could be attributed to several components of the structure; these components ought to be considered as "errant" ones. But it remains always necessary to place 6 *Al*- and 6 *Si*-atoms (or ions) within the elementary cell, which constituents probably cannot be considered as erratic ones. These atoms must, therefore, occupy

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[1]) W. L. BRAGG and Coll., Proceed. R. Soc. London *A*. **110**, (1926), 34; ibid., *A* **111**, (1926), 691; ibid., *A*, **113**, (1927), 642; Zeits. f. Kryst., **63**, (1926), 122, 538; etc.

[2]) Loco cit., These Proceed. **30**, (1927), 249.

either two different *six*-fold places, or they must be distributed over a single *twelve*-fold position, — which seems not so hazardous, if one takes into account the great analogy in function of the *Al*-, and *Si*-atoms with respect to their ROENTGEN-spectrographical behaviour. But unfortunately there is in none of the space-groups mentioned, which all correspond to the *body*-centred grating, a possibility to distribute the *Al*-, and *Si*-atoms over two different *six*-fold places.

However, it must be remarked, that the presence of the bodily-centred grating was deduced from the fact, that reflections of planes $\{hkl\}$, in which $(h+k+l)$ is *odd*, have never been observed: all reflections actually found are such, for which the sum of the indices is an *even* number. On the other hand, this fact can also be interpretated so, that the fundamental lattice is *not* the bodily-centred one, but that the special arrangement and the parameters of the atoms within the cell is such, that the odd orders of the diffraction-images of planes $\{hkl\}$, for which $(h+k+l)$ is odd, have such weak intensities, as to make them undetectable; also in this case the impression will be left, that the grating is a bodily-centred one. In the case of *ions* being the building-stones, these reflections might also be completely absent, as $Al^{\cdot\cdot\cdot}$ and $Si^{\cdot\cdot\cdot\cdot}$-ions have the same number of electrons.

It is for this reason, that a more promising way could perhaps be found by placing the atoms within the cell is such a way that all intensities for lines $\{hkl\}$, in which $(h+k+l)$ is odd, become so small that they are *practically* equal to *zero*. The choice of the space-group must, of course, be a rational one, in which the valencies of the different geometrical positions occurring in it, are in agreement with the number of atoms of the same kind to be placed into the cell.

For this attempt the space-group O_h^9 and its sub-groups: O_h^3 en O_2 on the one side and T_d^4, T_h^1 and T^1 on the other side, appear to be well suited. The first mentioned groups O_h^9 correspond to a bodily-centred, the second series of groups to a simple cubic lattice. The groups O_h^3, etc. present the advantage that the 6 *Si*-, and the 6 *Al*-atoms can be distributed over two different *six*-fold positions ($6f$ and $6g$ in WYCKOFF's notation[1]); in the case of *nosean* T_d^4 has some advantages, if also to the (SO_4)-groups of the Na_2SO_4 a "fixed" place shall be attributed. In choosing O_h^9 and its subgroups, the 6 *Al*- and 6 *Si*-atoms must be distributed over one *twelve*-fold place ($12h$ in WYCKOFF's notation); — as was already said, this is perhaps allowed for the calculation of the intensities with respect to the great analogy in diffracting power of both kinds of atoms, which in the case of their being present as *ions*, even becomes identical. As will soon become evident, however, the intensities calculated by the two methods, will appear to differ so insignificantly, when a suitable choice of the parameters is once made,

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[1]) R. W. G. WYCKOFF, *The Analytical Expression*, etc., p. 130; p. 148.

— that it even appears questionable whether it will ever be possible to discriminate between these space-groups, solely by means of the relative intensities actually observed. It will always be difficult to decide by the spectrographical method, whether the *Al*- and *Si*-ions occupy two *six*-fold positions, or one *twelve*-fold one, in which they can substitute each other isomorphously. It is very well possible that, on account of their different chemical characters, they yet occupy two *six*-fold places in the crystalline structure. The fact, that in the *ultramarines* the atomic ratio: $Al : Si$ is *not* constant is, as we shall see, rather an indication, however, that they really can occupy a *twelve*-fold position. It might be possible that, in some way, a gradual transition within this group of compounds from the simple-cubic into the bodilycentred grating, were present.

§ 3. The calculation of the intensities has now been made in such a way that the space-group T_d^4 was assumed to be present here. To the atoms present within the elementary cell of *nosean*: $Na_{10}Al_6Si_6O_{32}S_2$ the following positions were attributed, *two* of the *Na*-atoms (or ions) being considered to be errant ones [1]):

To the *S*-atoms: $(0\,0\,0)$ and $(\frac{1}{2}\,\frac{1}{2}\,\frac{1}{2})$.

To the 8 *Na*-atoms remaining:

$$(u\,u\,u),\ (\bar{u}\,u\,\bar{u}),\ (u+\tfrac{1}{2},\,u+\tfrac{1}{2},\,u+\tfrac{1}{2}),\ (\tfrac{1}{2}-u,\,u+\tfrac{1}{2},\,\tfrac{1}{2}-u),$$
$$(u\,\bar{u}\,\bar{u}),\ (\bar{u}\,\bar{u}\,u),\ (u+\tfrac{1}{2},\,\tfrac{1}{2}-u,\,u+\tfrac{1}{2}),\ (\tfrac{1}{2}-u,\,\tfrac{1}{2}-u,\,u+\tfrac{1}{2}).$$

To the 6 *Al*-atoms:

$$(0\,\tfrac{1}{2}\,\tfrac{1}{4}),\ (\tfrac{1}{4}\,0\,\tfrac{1}{2}),\ (\tfrac{1}{2}\,\tfrac{1}{4}\,0),\ (0\,\tfrac{1}{2}\,\tfrac{3}{4}),\ (\tfrac{3}{4}\,0\,\tfrac{1}{2}),\ (\tfrac{1}{2}\,\tfrac{3}{4}\,0).$$

To the 6 *Si*-atoms:

$$(\tfrac{1}{2}\,0\,\tfrac{1}{4}),\ (\tfrac{1}{4}\,\tfrac{1}{2}\,0),\ (0\,\tfrac{1}{4}\,\tfrac{1}{2}),\ (\tfrac{1}{2}\,0\,\tfrac{3}{4}),\ (\tfrac{3}{4}\,\tfrac{1}{2}\,0),\ (0\,\tfrac{3}{4}\,\tfrac{1}{2}).$$

To the 8 *O*-atoms the (SO_4)-groups:

$$(v\,v\,v),\ (\bar{v}\,v\,\bar{v}),\ (v+\tfrac{1}{2},\,v+\tfrac{1}{2},\,v+\tfrac{1}{2}),\ (\tfrac{1}{2}-v,\,v+\tfrac{1}{2},\,\tfrac{1}{2}-v),$$
$$(v\,\bar{v}\,\bar{v}),\ (\bar{v}\,\bar{v}\,v),\ (v+\tfrac{1}{2},\,\tfrac{1}{2}-v,\,\tfrac{1}{2}-v),\ (\tfrac{1}{2}-v,\,\tfrac{1}{2}-v,\,v+\tfrac{1}{2}).$$

To the 24 other *O*-atoms:

$$(x\,y\,z),\ (x\,\bar{y}\,\bar{z}),\ (\bar{x}\,y\,\bar{z}),\ (\bar{x}\,\bar{y}\,z),\ (y+\tfrac{1}{2},\,x+\tfrac{1}{2},\,z+\tfrac{1}{2}),\ (\tfrac{1}{2}-y,\,x+\tfrac{1}{2},\,\tfrac{1}{2}-z),$$
$$(y+\tfrac{1}{2},\,\tfrac{1}{2}-x,\,\tfrac{1}{2}-z),\ (\tfrac{1}{2}-y,\,\tfrac{1}{2}-x,\,z+\tfrac{1}{2}),$$
$$(z\,x\,y),\ (\bar{z}\,x\,\bar{y}),\ (\bar{z}\,\bar{x}\,y),\ (z\,\bar{x}\,\bar{y}),\ \text{en}\ (x+\tfrac{1}{2},\,z+\tfrac{1}{2},\,y+\tfrac{1}{2}),\ (x+\tfrac{1}{2},\,\tfrac{1}{2}-z,\,\tfrac{1}{2}-y),$$
$$(\tfrac{1}{2}-x,\,\tfrac{1}{2}-z,\,y+\tfrac{1}{2}),\ (\tfrac{1}{2}-x,\,\tfrac{1}{2}+z,\,\tfrac{1}{2}-y),$$
$$(y\,z\,x),\ (\bar{y}\,\bar{z}\,x),\ (y\,\bar{z}\,\bar{x})\ (\bar{y}\,z\,\bar{x}),\ (z+\tfrac{1}{2},\,y+\tfrac{1}{2},\,x+\tfrac{1}{2}),\ (\tfrac{1}{2}-z,\,\tfrac{1}{2}-y,\,x+\tfrac{1}{2}),$$
$$(\tfrac{1}{2}-z,\,y+\tfrac{1}{2},\,\tfrac{1}{2}-x),\ (\tfrac{1}{2}-z,\,y+\tfrac{1}{2},\,\tfrac{1}{2}-x).$$

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[1]) In a rigorously geometrical lattice, Na_2SO_4 in *nosean* and *hauyne* cannot be substitued by $CaSO_4$, nor in the *ultramarines* by Na_2S_2, Na_2S_3, etc. From these facts alone it appears inevitable to give up the supposition of a rigorously geometrical arrangement of these components within the whole structure. Also the previous argument for the necessity of assuming the presence of such "erratic" constituents must be kept in mind in this connection.

Experience showed that the most suitable values of the parameters were: $u = 0.250$; $v = 0.103$; $x = o$; $y = z = 0.354$.

The value: $u = {}^1/_4$ causes the differences of the results to be annihilated as concerns the *Na*-ions, in the case that O_h^3 or that T_d^4 are chosen.

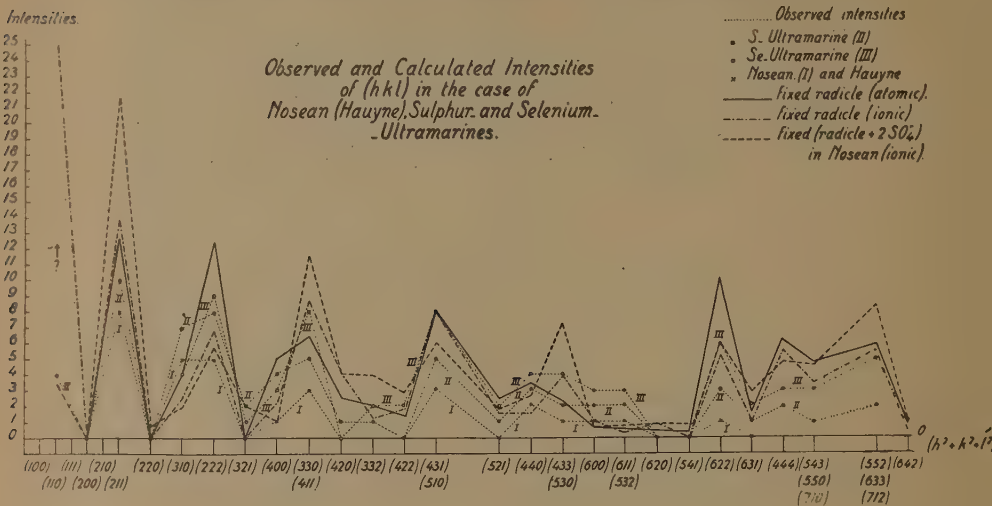

Fig. 2.

In figure 2 the calculated intensities are graphically represented, as well in the case that the "fixed" constituent: ($Na_8Al_6Si_6O_{24}$) is considered alone, be it as built-up of "atoms" (full-drawn line), or of "ions" (broken line), — as in the case that also fixed positions are attributed to the (SO_4)-groups (dotted-point-line); the observed intensities of *nosean* and of the *S*-, and *Se-ultramarines* are represented by the dotted lines I, II and III. The agreement between observed and calculated values appears to be a very satisfactory one, the more so, when it be remembered how subjective and rough the visual estimation of the intensities of the diffraction-images always is. Moreover, the DEBEIJE-factor causes the images of diffraction with a great value of the angle of deviation to get relatively lower intensities then the others. For this reason the apparently better agreement between observed and calculated intensities of the "fixed" radicle, if it be supposed to be built-up by atoms, instead of by ions — cannot be considered as a final argument against the presence of an ionic lattice: the fact mentioned only indicates that the diffractive power of an O''-ion is somewhat smaller than that of an $Al^{\cdot\cdot\cdot}$-, or a $Si^{\cdot\cdot\cdot\cdot}$-ion. In every case it appears that the curve of the intensities for the radicle + two (SO_4)-groups also considered as "fixed" constituents, does not agree so well with the observations as in the case, that these

(SO_4)-groups are errant ones. But even in this case, the differences are only insignificant, if the uncertainty of visual estimation of the intensities is taken into account. *It is evident, however, that the characteristic* ROENTGEN-*spectrogram of nosean (hauyne) and of the ultramarines is completely determined by the diffraction-image caused by the radicle:* $(Na_8Al_6S_6O_{24})^{\cdot\cdot}$ *with the structure indicated in the above.* Characteristic is the absence of the images (200) and (220), and also, that (211) yields the most intensive diffraction-image. The estimation of the intensity of (110) is very uncertain, because this image is always situated within the field of heaviest blackening of the film and within the central halo, — it being, therefore, hardly discernible; in the few cases that it could be observed, its intensity appeared to be rather appreciable. The line (420) has been observed on some spectrograms, in the immediate neighbourhood of (330).

It may here be remarked once more that the arrangement of the "fixed" radicle as described above, can as well occur by means of the space-groep O_h^3 and its (geometrical) sub-groups O_2, T_d^4 T_h^1 and T^1. If, however, in *nosean* also to the (SO_4)-groups *fixed* places are attributed, the holohedral symmetry can by no means be maintained, and the group T_d^4 is then the most probable one.

§ 4. The structure of *nosean* as deduced before, has in Fig. 3 schematically been represented as a projection upon (001).

The *Al*-, *Si*-, and *S*-ions are all situated within the lateral faces of the elementary cell. The four O-ions in the centre of the cell are on *zero*-level, those to the left on $^1/_2$ of the cube-edge. The (SO_4)-group in the middle of the cell (in $^1/_2$ of the height of the cube) has, for the sake of clearness been omitted. To the right the (SO_4)-groups are represented, which are also visible through the hole in the centre of the basal plane and to the left in the corners. The *sodium*-ion has much free space with respect to the surrounding 6 *O*-ions, and may occasionally be substituted by elements having a greater atomic radius.

In the corners of the cell and in the centre of the basal face binary axes of rotation are present perpendicularly to (001); so are quaternary mirror-axes in the middle of the edges. The diagonals of the basal face are the directions of intersection with it of translatory mirror-planes, the number of them being twelve pro elementary cell. The body diagonals are ternary axes of rotation; moreover, a set of eight trigonal helicoidal axes is present, which axes are parallel to the first mentioned ones, for instance: $[111]_{0,0}$; $[111]_{1/_3,2/_3}$; $[111]_{2/_3,1/_3}$; etc.

Two cases are to be distinguished:

a. If only $(Na_8Al_6Si_6O_{24})^{\cdot\cdot}$ is present as the "fixid" part (full-drawn line in Fig. 2), it appears:

1. That in each lateral face 2 *Al*-, and 2 *Si*-ions are situated, each

of them being surrounded by 4 O-ions; each of them has in T_d^4 the proper symmetry S^4. (In O_3^h, however, the symmetry V_D).

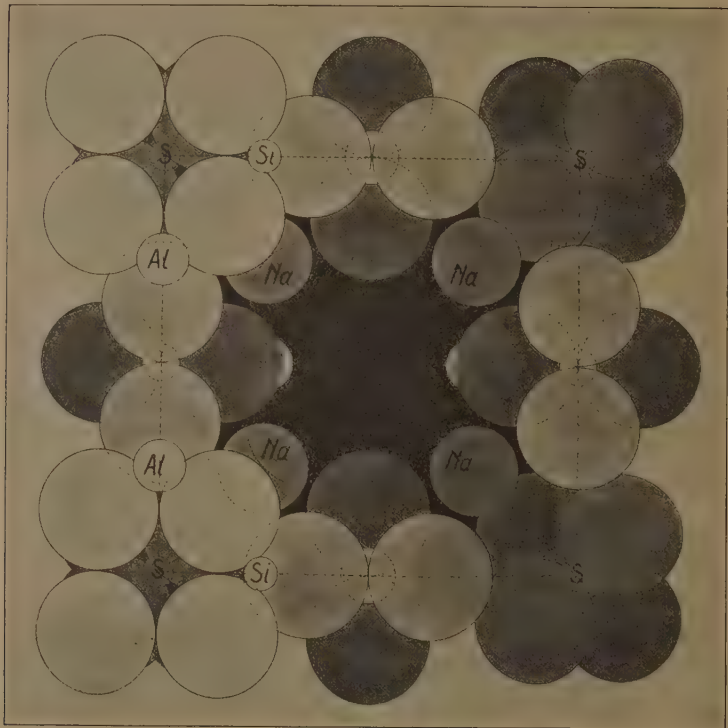

Fig. 3.

The Structure of Nosean and of the fixed radicle in Ultramarine. The lower part of the elementary cell has been projected upon (001): to the right only the basal layer, to $^1/_4$ of the height, — to the left to $^1/_2$ of the height of the cube. Hence, in the figure only that part of the contents of the cell is indicated which is situated beneath and in the plane (012). The fig. spheres represented the O-ions; etc.

2. That the 8 *Na*-ions are situated upon the four *body* diagonals, at $\frac{1}{4}$ and $\frac{3}{4}$ of their length; each of them is, at a distance of 2.64 A.U., surrounded by 6 *O*-ions. Their proper symmetry is D_3 in the space-group O_h^3, in T_d^4, T_h^1 and T^1, however, C_3 (in T^1 distributed over 2 groups of 4 *Na*-ions). Without giving up the supposition of tetrahedrally arranged *O*-ions, it is impossible to place 6 of the 8 *Na*-ions into the middle of the lateral faces and into the middle of the edges, as ought to be the case in the only *six*-fold place present within the cell of the bodily-centred grating previously indicated. The space left free within

the interstices mentioned is namely no more than 0.54 A.U., the radius of the *Na*-ion being already 0.99 A.U.

Instead of the *Na*-ions, into the space between the 6 *O*-ions ($r = 1.33$ A.U.), $Li^{\cdot}$-, $K^{\cdot}$- and $(NH_4)^{\cdot}$-ions, and also $Ag^{\cdot}$-ions can easily introduced, the radii of which have the same or analogous size[1]; but less easily $Rb^{\cdot}$-, and still less $Cs^{\cdot}$-ions can be substituted, — a fact which is also in agreement with the result previously published concerning the replaceability and the velocity of substitution of the *Na*-ions in the *ultramarines* by the interaction of the salt-solutions of these metals.

The 8 *Na*-ions, which are situated near two great cavities of the structure, at a distance from the vicinal *O*-ions that is greater than the sum of the radii of these ions, are, indeed, highly mobile ones. It is very probable that the character of the *permutites* which the *ultramarines* possess and which is exactly determined by the easy replaceability of those *Na*-ions by other mono- or bivalent metallic ions, is intimately connected with this freedom of motion within the structure proposed.

3. That the large cavity in the middle of the elementary cell (cubical space with $a_0 = 3.7$ A.U. and three cross-wise arranged, perpendicular prolongations) offers plentiful space to the negatively charged groups: $NaSO'_4$, NaS'_3 or $(Na, Ca)\,SO'_4$, which there and in the corners must be present as errant substitutes; they are kept electrostatically linked by the positively charged fixed part of the structure. The cavity in the middle and those in the corners of the cell can serve as a receptacle for these constituents, which in the case of *nosean* can be considered either as fixed or as errant ones, but in the case of *ultramarines* only as of the last kind, the place of which is not determined by the special symmetry of the total geometrical arrangement.[2] A true

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[1]) The radii of these and other ions, which, in the course of these investigations were introduced into the *ultramarines*, have the following values:

$Li^{\cdot}$ 0.79 A.U.	$Rb^{\cdot}$ 1.49 A.U.	Moreover: $Zn^{\cdot\cdot}$ 0.83 A.U.
$Na^{\cdot}$ 0.99 A.U.	$Cs^{\cdot}$ 1.65 A.U.	$Ca^{\cdot\cdot}$ 1.06 A.U.
$K^{\cdot}$ 1.33 A.U.	$Tl^{\cdot}$ 1.49 A.U.	$Sr^{\cdot\cdot}$ 1.27 A.U.
	$Ag^{\cdot}$ 1.13 A.U.	$Ba^{\cdot\cdot}$ 1.42 A.U.

$Mg^{\cdot\cdot}$ 0.78 A.U.	$Al^{\cdot\cdot\cdot}$ 0.57 A.U.	O'' 1.33 A.U.
$Hg^{\cdot\cdot}$ 1.12 A.U.	$Si^{\cdot\cdot\cdot\cdot}$ 0.39 A.U.	Se'' 1.91 A.U.
$Pb^{\cdot\cdot}$ 1.32 A.U.	S^{VI} 0.34 A.U.	For the *S-atom* is: $r = 1.04$ A.U.
$Mn^{\cdot\cdot}$ 0.91 A.U.	S'' 1.74 A.U.	For the *Se-atom* in the same way: 1.13 A.U.

For the purpose of giving a place to the *Si*-, *Al*- and S^{VI}-ions within the tetrahedral groups of 4 *O*-ions, to them diameters must be attributed, which are not greater than 0.58 A.U.

[2]) It may be remarked, that in *nosean* only 2 *Na*-atoms, or perhaps also the two (SO_4)-groups are erratic constituents; but in the *ultramarines* only the 2 *S*-atoms of those poor in *silica* and of a low degree of sulphuration (with S_2) can be placed into the structure. If S_3 or S_4 is present in them, these *sulphur*-atoms as well as the *Na*-atoms in excess, must all be errant ones.

"mother-silicate" is therefore, *not* present in the *ultramarines*; the underlying "fixed" component is, in a certain way, comparable to an electrically charged *radicle:* $(Na_8Al_6Si_6O_{24})^{\cdot\cdot}$. Hence, the question as to the isolation of the underlying component, common to all these minerals, appears to be idle, in the same way as that of isolating the $(NH_4)^{\cdot}$-ion from the *ammonium*-salts. The 2 *Na*-atoms of the ten atoms present in *nosean*, which are freely movable within the large cavities mentioned above, will, in the beginning, more easily be substituted than the others by *Ag* or other metals, and this substitution will only have a feeble influence upon the relative intensities of the diffraction-lines; but such an influence may be beforehand expected as soon as also *Na*-atoms have gradually been replaced by *Ag*, so that this now occupies also places within the fixed part of the structure. Also this fact was previously stated by us; only after a certain percentage (more than 12—18 %) of the *Ag* has been introduced instead of the *Na* present, the powder-spectrograms begin to lose the character of the *sodium-ultramarines* so as to change into that of the *silver-ultramarines*.

b. If the supposition is made, — as was done in the calculation, — that besides $(Na_8Al_6Si_6O_{24})^{\cdot\cdot}$ also both (SO_4)-groups have a "fixed" place in the structure, only 2 *Na*-ions still being erratic constituents, — then it becomes necessary to place those (SO_4)-groups into the centre and the corners of the elementary cell in such a way, that the 4 *O*-ions are again arranged tetrahedrally and in the right position with respect to their surroundings; i.e. on the *body* diagonals and contiguous to the *Na*-ions. The symmetry in this case can no longer be O_h^3, but surely T_d^4, O_2, T_h^1 and T^1. In the space-groups T_d^4 and T^1 the ternary axes are heteropolar; cube-diagonals are occupied in such a way that each *Na*-ion is situated between two *S*-atoms in the positions (000) and $(\frac{1}{2}\frac{1}{2}\frac{1}{2})$ at a certain distance from them or occasionally also exactly midway between them; each *S*-atom is surrounded by four tetrahedrally arranged *O*-ions and the distance of each *Na*-ion to the top of the subsequent tetrahedron of *O*-ions is either greater or smaller than that to the basal face of the preceding tetrahedron, etc. In the space-group T_d^4 each *Na*-ion has one variable parameter u, so that it can be shifted along a ternary axis, for instance into the direction of a corner or away from it. If this parameter u has exactly the value necessary to make it situated just in the middle of two subsequent *S*-atoms, then its distance to the top of the *O*-tetrahedron immediately following will be about 2.30 A.U.; this value is very close to that of the sum of the radii of a contiguous $Na^{\cdot}$- and O''-ion, namely: $(0.99 + 1.33) = 2.32$ A.U. In the space-group T_h^1 the ternary axes are no longer heteropolar; the subsequent tetrahedra of *O*-ions have, in this case, alternately reversed positions, so that each *Na*-ion is alternately situated either between two basal faces or between two tops of such tetrahedra. However, in this case the diffraction-images of faces (hkl), for which $(h+k+l)$ is odd,

would yet appear to have detectable intensities, so that it must be concluded that the presence of this space-group T_h^1 is rather improbable.

§ 5. As is well-known, chiefly two series of *ultramarines*, those poor and those rich in *silica*, are distinguished in industry, the ratio: $Al:Si$ in the first being: $6:6=1:1$, in the last: $4:6=1:1.5$. Experience, however, teaches us, that:

1. All these *ultramarines* yield very *in*constant results on analysis, so that it becomes certain, that their composition is *not* constant and can only approximately be expressed by definite stoechiometrical formulae;
2. All these *ultramarines* give *identical* powder-spectrograms with only slight and hardly detectable differences of their relative intensities; moreover, their fundamental gratings have practically identical dimensions;
3. They can all be separated by means of repeated decantions into fractions of different, be it only very little different, specific gravities. As these substances have towards water, however, highly troublesome (hydraulic) properties, it is practically impossible to separate from them fractions of definite specific weight to any satisfactory degree.

The thought arises from all these facts, that one has to deal here with *mixtures* of analogous, *isomorphous* substances, and that this is the true cause as well of their *in*constant composition, as of the identity of their ROENTGEN-spectrogram.

According to what has been said in the above about their probable structure, the sum of the number of their *Al*- and *Si*-ions will be *twelve*. As only inappreciable differences of the diffractive power exist in the case of an *Al*- or a *Si*-atom and these differences are even zero in the cases of $Al^{\cdots}$ and $Si^{\cdots\cdot}$-ions, — also *ultramarines*, in which the place of an *Al*-atom is occasionally occupied by a *Si*-atom [1]) and vice-versa, will show a distribution of their intensities, which practically cannot be distinguished from that of a *silicate* having a fixed part with the ratio: $Al:Si=6:6$. Thus, for instance, *ultramarines*, in which the „fixed" radicles are:

$$(Na_8Al_5Si_7O_{24})^{\cdots}$$
$$(Na_8Al_4Si_8O_{24})^{\cdots\cdot} \text{ etc.,}$$

will all show powder-spectrograms, which are practically *identical* with that of the *ultramarine* poor in *silica*. It must then be evident, however, that the *mixtures* of such isomorphous *ultramarines* can very well show empirical chemical compositions, which very closely approach to that of an apparent "compound", in which the ratio $Al:Si$ is about $1:1.5$. Thus, for instance, a mixture of 2 "mol" of an *ultramarine* with $(Na_8Al_4Si_8O_{24})^{\cdots\cdot}$ as fixed radicle, and 1 "mol" of one containing as

[1]) It must be remarked in this connection, that in 1910 J. HOFFMANN already drew attention to the fact, that the quantity of *boron* found by him in the completely *silica-free boron-ultramarines*, exactly corresponds to *the sum* of all *Si*-, and *Al*- of the ordinary *ultramarines*; so that it seems able to substitute as well the congenial *Al*-ion, as the *Si*-ion.

such the radicle: $(Na_8Al_6Si_6O_{24})^{\cdot\cdot}$, corresponds to an empirical composition of an apparent "mol" with the radicle: $(Na_8Al_{4.6}Si_{7.3}O_{24})$, in which the ratio: $Al:Si$ will be: 1 : 1.57, etc. It is also very well possible that the symmetry of the corresponding space-group is somewhat lowered by such substitutions, for instance to that of one of the other space-groups. But, as was already mentioned before, even this circumstance will hardly, if at all, be detectable by the relative intensities calculated. From all this it becomes clear, that it is no use attributing a definite chemical formula to any species of *ultramarine*, only on the ground of the results of analysis accidentally found. For quite independently of the way in which the substitution of Al by Si, or reversily, has taken place, — the powder-spectrograms, remain practically indentical, — even if the erratic constituents of the structure: NaS'_2, NaS'_3, NaS'_4, $NaS_2O'_3$, etc., be replaced by others [1]).

Another fact may still be brought to the fore here: in the *ultramarines* with S_2 as well as with S_3 or S_4, the *sulphur*-atom surely is *not* hexavalent, as was supposed to be the case in the (SO_4)-groups of *nosean*; but it is, at the highest, *bi*valent. But the radius of the S''-ion is: 1.74 A.U., hence its diameter: 3.48 A.U. Therefore, 2 S''-ions can surely replace the total (SO_4)-groups of *nosean*; but as the last have a greater volume, the 2 S''-ions will have considerable space to move freely. In the *ultramarines* of higher degrees of sulphuration, — which, like the *polysulphides*, are the more intensely coloured, the higher their *sulphur*-content is, — the whole remaining part of the *sulphur* must, under all circumstances, therefore, certainly be "erratic". But will this be in the form of ions or atoms? We do not know; perhaps that *S-atoms*, if dispersed in a medium built-up by electrically-charged *ions*, may cause the occurrence of the blue coloration, as this has been observed also in the case of other systems containing dispersed sulphur [2]).

§ 6. Also the absorption-phenomena described previously in this paper become more comprehensible now, when considered in the light of these conceptions about the structure of the *ultramarines*. For the fixed component, consisting of the doubly electrically-charged radicle: $(Na_8Al_6Si_6O_{24})^{\cdot\cdot}$ or of others, differing from it only by the partial substitution of Al by Si and reversily, — represents the proper absorbent crystalline skeleton of all these compounds; hence, it is conceivable that the surface of the *ultramarine*-granules is highly "activated" and that

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[1]) The presence of a radicle: NaS_2O_3' remains as yet hypothetical, as long as no *thiosulphate* has directly been separated from *ultramarines*: it is equally well possible, that a mixture of S' and $(SO_4)''$ forms the errant components.

[2]) J. HOFFMANN, loco cit.; F. KNAPP, Dingler's Polyt. Journ., **233**, (1879), 479; P. VON, WEIMARN, Koll. Beihefte, **22**, (1926), 38; WO. OSTWALD, ibid., **2**, (1911), 409, J. HOFFMANN, Koll. Zeits., **10**, (1912), 275; PATERNÒ and MAZZUCHELLI, Atti R. Acad. d. Lincei, (5) **16**, I, (1907), 465.

the molecules of the solutions, brought into contact with them and entering the large cavities of their structure, will be easily fixed upon these grains, especially after the solvent has taken up a part of the errant constituents within those cavities. It would, for instance, not be at all absurd to suppose, that the negative *O*-ions which are situated in the walls of the central cavity, subsequently would help to fix upon them the cations of the molecules already absorbed in this way. It must be remarked, moreover, that the surface of these *ultramarine*-granules with their characteristic "honeycomb"-structure and their cavities, is essentially different from the architecture of an atomic plane in a crystal-species like *sodiumchloride*. This loose structure, characterized by a relatively great specific surface, probably causes those "surface-actions" to come more particularly to the fore. Subsequently a gradual substitution of the *sodium*, — in the first place of the erratic *Na*-atoms and afterwards also of the *Na*-atoms (or ions) in the fixed radicle, — will possibly take place. The great velocity of the process in the beginning, is probably related to such an absorption of the salt-solution in the cavities and the loosening and substitution of the erratic constituents in them, while the farther, more intrinsic substitution then more particularly corresponds to the flatter part of the curve in Fig. 1. It is even most probable that the "hydraulic" properties of the *ultramarines* are also connected with those facts: the molecules of H_2O also will penetrate into the said cavities and evidently they too will readily settle down upon the electrically charged surface of the structural skeleton, after the errant components have been removed.

§ 7. After all that has been said it must have become clear now, that all attempts to express the constitution of these minerals and more particularly that of the *ultramarines* are to be considered as failures and that they inevitably were predestined to be so. The same is true for those, in which it is tried to express that constitution by means of WERNER's views about co-ordinative arrangement[1]) and the *ultramarines* are considered as derivatives of a complex ion: $\{Al(SiO_4)_3\}^{IX}$. In general, such *silicates* in the solid state only appear to be regular aggregations of atoms or ions, in which more particularly the *oxygen*-ions have a predominant part. There is, however, no special reason to assume a stronger link between some of them, than between others, — with perhaps the exception of such groups as (SiO_3), (SiO_4), etc. The principle according to which *Al*- and *Si*-atoms may mutually replace each other within such *silicate*-structures as those of *alumo-silicates*, — as both kinds of atoms only differ by the possession of a single electron less or more, — without any appreciable variation of the general character of the structure itself, seems to us to be a sound one and to be of importance with respect

[1]) Conf. i.a. J. JACOB, Zeits. f. anorg. Chem., **106**, (1919), 229; Helv. Chim. Acta, **3**, (1920), 669; E. GRUNER, Zeits. f. angew. Chem., (1928), 447, 448.

to the future elucidation of the structures of other solid *silicates*; and this is even more probable in the case that $Al^{\cdots}$- en $Si^{\cdots\cdot}$-ions play a part, both having the configuration of *neon*. The same is true with respect to the supposition made about the possible occurence of errant constituents in such *silicates*, which components may be intercalated within the void spaces between the fixed parts of the whole structure; and in the first place we remember, in this connection, of the *zeolites*.

§ 8. By the aforesaid investigations the more than a hundred years old problem concerning the constitution of the *ultramarines*, etc., is certainly brought appreciably nearer to its solution, but yet we are far from a final decision. In the first place it will be necessary to check the structure developed in this paper also in those cases in which *sodium* is substituted by $Ag^{\cdot}$, $Li^{\cdot}$, $Rb^{\cdot}$, $Cs^{\cdot}$, $Tl^{\cdot}$-ions or by those of $Ca^{\cdot\cdot}$, $Ba^{\cdot\cdot}$, $Sr^{\cdot\cdot}$, $Pb^{\cdot\cdot}$, $Hg^{\cdot\cdot}$ etc.

This includes still a great number of tedious and lengthy calculations of the intensities, the more so, as for instance the 8 *Na*-ions will be replaced by 4 probably tetrahedrally arranged ions of the bivalent metals mentioned, and occasionally even other space-groups ought to be taken into account then. Moreover, it will be necessary still to do much analytical work with the purpose of ascertaining whether in the substitution-products obtained in the various reactions there is still *Na* present and to what amount; and also, whether *Al*- or *Si*-atoms are removed, as was the case in the transformation of GUIMET's blue into *silver-ultramarine*. Not until all such questions shall have been answered will the structure of the *ultramarines* be finally known.

Even then the explanation of the intense and stronge coloration-phenomena of the *ultramarines* remains to be given. That they depend on the errant constituents: in the first place on the presence of *S*-atoms or S''-ions, or NaS'_n-ions respectively, — is certain. The blue colour increases with the *sulphur*-content, -exactly as the colour of the *poly-sulphides* in general appears to get deeper, as they contain more *sulphur*; but at the same time, this colour is also a function of the metal present: $Na^{\cdot}$-, $Li^{\cdot}$-, $Tl^{\cdot}$-, $Ag^{\cdot}$-, $Hg^{\cdot}$-*ultramarines* on the one hand, $Ca^{\cdot\cdot}$-, $Sr^{\cdot\cdot}$-, $Ba^{\cdot\cdot}$-, $Zn^{\cdot\cdot}$- and $Pb^{\cdot\cdot}$-*ultramarines* on the other hand, can demonstrate the truth of this. Also the example of the *boron-ultramarines* which contain neither *Si*, nor *Al*, and in which, according to HOFFMANN, the blue colour *suddenly* appears, as soon as the molten mass or the solid phase has been brought into definite physical conditions, — always contain a certain amount of an electropositive metal like *sodium*.

The blue colour of systems containing *sulphur* seems to be connected with *dissociation*-, or, at least, with *dislocation*-phenomena, occurring in the *sulphur*-compounds used. Thus, for instance, *potassiumrhodanide* becomes quite blue if melted; but this blue colour again disappears when the molten mass is cooled. This phenomenon must be connected in some

way with a reversible transformation, for instance, with a reaction of the form: $KCNS \rightleftarrows KCN + S$ or $\rightleftarrows K^{\cdot} + CN' + S_{(act)}$. Indeed, *sulphur* introduced into molten *KCN* appears to behave in a similar way. It would be interesting in this respect to study the optical properties of finely dispersed *sulphur* in fields of strong electrostriction, as here a physical phenomenon presents itself, which is still unexplained. Also in this respect many investigations will still have to be made, before the mystery of the *ultramarines* can be said to be definitively solved.

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Anatomy. — *The fissures on the frontal lobes of Pithecanthropus erectus Dubois compared with those of Neanderthal men, Homo recens and Chimpanzee.* By C. U. ARIËNS KAPPERS.

(Communicated at the meeting of December 22, 1928).

Brief descriptions of the frontal fissures on the endocranial cast of Pithecanthropus erectus are given by DUBOIS himself [1]) and by KEITH [2]), along with some remarks in TILNEY's and RILEY's book [3]).

Of the illustrations hitherto given those of KEITH are the most instructive but also his description is not complete. Also the lunate sulcus may be observed, especially on the right occipital lobe, where the ram. posterior calcarinae seems to be indicated, together with a slight impression behind it, caused perhaps by the posterior limb of ELLIOT SMITH's superior occipital furrow (Ypsiliformis mihi, triradiatus LANDAU). On the left hemisphere the sulc. lunatus cannot be recognized with equal probability.

As far as concerns the central sulcus I can only give a supposition regarding its ventral ending (see below).

The frontal fissures, however, throw a very interesting light on this object.

Starting with the *right frontal lobe* I shall indicate its fissures by figures in order to avoid the precocious homologies, necessarily included in names.

I first call attention to the incision of the orbital margin of the frontal lobe (1, fig. 1). This indentation continues backwards underneath the orbital operculum and forms its ventral border. The upper limit of this operculum is indicated by fissure 2. On the operculum orbitale itself an axial groove 3 appears. This has only a short frontal course connecting in the middle with the curved fissure 4, that encircles fissure 2 and then, — after giving off a small caudo-ventral branch 4^1 continues with a new curve in the slightly oblique X shaped fissure 5. Where the latter furrow ends with a caudo-dorsal branch 5*a*, a strong frontal fissure 6 proceeds from it in the direction of fissure 7. This fissure 7 has a strong caudal

1) DUBOIS. Remarks upon the braincast of Pithecanthropus erectus: Proc. of the 4th internat. Congress of Zoology Cambridge, 1898. DUBOIS. On the principal characters of the cranium and the brain etc. of Pithecanthropus erectus. Proceed. of the Kon. Akad. van Wetenschappen, Amsterdam, Vol. 27, 1924, N⁰. 3 and 4; for figures see ibidem N⁰. 5 and 6.

2) Report on the Galilee skull. Publ. of the Brit. school of Archeology in Jerusalem, London, 1927.

3) TILNEY and RILEY. The brain from Ape to Man. HOEBER, New-York, 1928.

branch *7a* [1]) issuing from it where it is connected with 6. Frontally, fissure 7 has two medio-dorsal branches *7b* and *7e*. The branch *7b* shows a bifurcation, the frontal limb of which establishes a connection with 11*c*, while its caudal limb proceeds in the direction of 11*b* without, however, reaching this branch.

At the frontal end of 7 two ventral branches occur, *7c* and *7d*. The connection of *7d* with 7 is only a superficial one, but is nevertheless clearly expressed. At its ventral end *7d* is very deep.

Between 7, 4 and 5 an *intermediate fosset* 8 occurs. On the right hemisphere this fosset is entirely independent, having no connection whatever with any of the surrounding fissures.

Frontally and ventrally of 7 lies fissure 9 that disappears under the fronto-orbital margin (the cast ends at the dotted line).

Finally, in front of 9 a shallow dimple, 10, occurs, apparently related to the border of the orbital rostrum.

On top of this whole system groove 11 appears, beginning caudally as a vague impression, the caudo-dorsal surface of the lobe being nearly flat. At 11*a* it has a small ventral offshoot, then another one, 11*b*, running in the direction of the posterior bifurcation of *7b* without reaching it, while 11*c* is distinctly connected with *7b*. Opposite 11*c* a dorso-medial offshoot

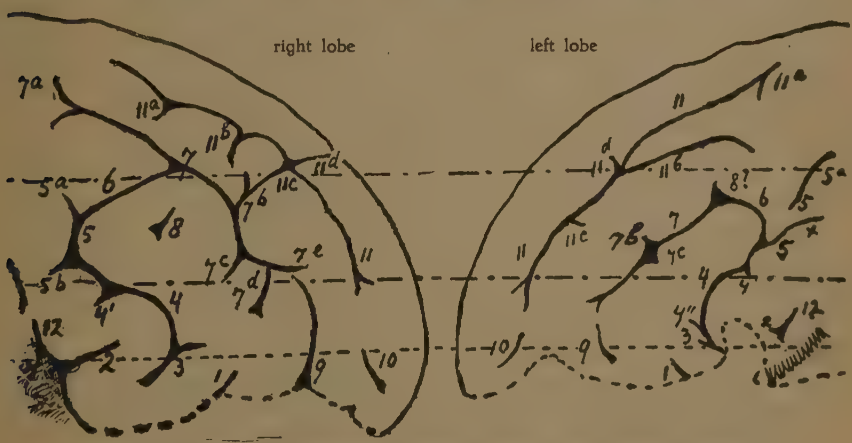

Fig. 1. The fissures on the frontal lobes of Pithecanthropus erectus Dubois.

11*d* reaches the medial hemispherial wall. Frontally, fissure 11 ends in a small bifurcation.

This is the structure of the right frontal lobe, to which I have only to add that at the posterior border of the lobe a fissure 12 occurs proceeding in the direction of the fossa Sylvii in which also 2 disappears. On top of 12,

[1]) The caudal bifurcation of this posterior branch *7a* is perhaps the beginning of a superior precental sulcus (see below p. 190 and fig. 4).

slightly behind it, a shallow deepening occurs (see fig. 1 behind 5*b*), not numbered in my figure, representing perhaps the ventral end of the centralis.

The fissuration on the *left frontal lobe* is equally as simple, as on the right. It is even slightly simpler in so far as the curve of 4 is somewhat steeper. For the rest its fissuration much resembles the one on the right lobe. As only a small part of the orbital operculum is left on this side, groove 1 is hardly expressed, and its caudal continuation under the operculum is missing.

Fissure 2 is perhaps indicated by a small frontal offshoot of a dimple, the larger part of which (12) runs dorsally. The axial groove of the orbital operculum 3 is indicated and connects here also with 4, the curve of which runs slightly steeper than on the right. Somewhat more dorsally from this curve a small offshoot proceeds frontally, 4″, which is missing on the right lobe. Dorsocaudally fissure 4, after making a new curve, connects directly with the lower end of fissure 6, fissure 5 being only indicated by a dorso-caudal groove 5*a*, which proceeds a little further in a dorso-caudal direction than indicated in my textfigure (cf. the plate). In the transverse fissure 5* a part of 5 is also included. The curved fissure 6 runs less dorsally on the left lobe than on the right one and an independent *intermediate fosset* 8 is failing on the left lobe.

In his interesting researches on the frontal lobe S. SERGI [1]) has frequently pointed out the various ways in which interfissural fossets behave, being sometimes taken up in adjacent grooves. As groove 7 on the left hemisphere runs more ventrally than on thé right it is not impossible that this fosset 8 is assimilated here with 7 (8? fig. 1).

Fissure 7 has only a slight indication of one ventral offshoot, which might correspond with 7*c* on the other side. Frontally, 7 ends in a bifurcation.

The sulcus 11 lying on top of this system differs somewhat from 11 of the other side in giving off a large caudoventral branch 11*b*. This may be explained by the more ventral course of 6 and 7 (see above). Opposite to 11*b* a small offshoot runs medially (11*d*), but does not reach the mesial margin of the hemisphere.

At the ventral margin of the left frontal lobe again the sulci 9 and 10 are indicated. Of these fissures 9 proceeds in the direction of 7.

Finally, this lobe also shows a shallow dimple behind 5* (not indicated in my figure), located, as its right homologue, immediately in front of the anterior branch of the arteria meningea media (see VLASSOPOULOS' drawings).

If I now compare these grooves with those that may be made visible by projection shadow and controlled by touch on the endo-cranial casts of Neanderthal men [2]) (fig. 2 and 3) it appears that their fissuration, though

[1]) S. SERGI. Sulle variazioni dei solchi del lobo frontale negli Hominidae. Rivista d'Anthropologia, Vol. 18, 1913.

[2]) For this purpose I used the endocranial casts of the Düsseldorf, La Chapelle, La Quina and Rhodesia man. The fissures on my endocranial cast of the Gibraltar woman are not distinct enough for this purpose. For these I refer to Sir ARTHUR KEITH's Antiquity

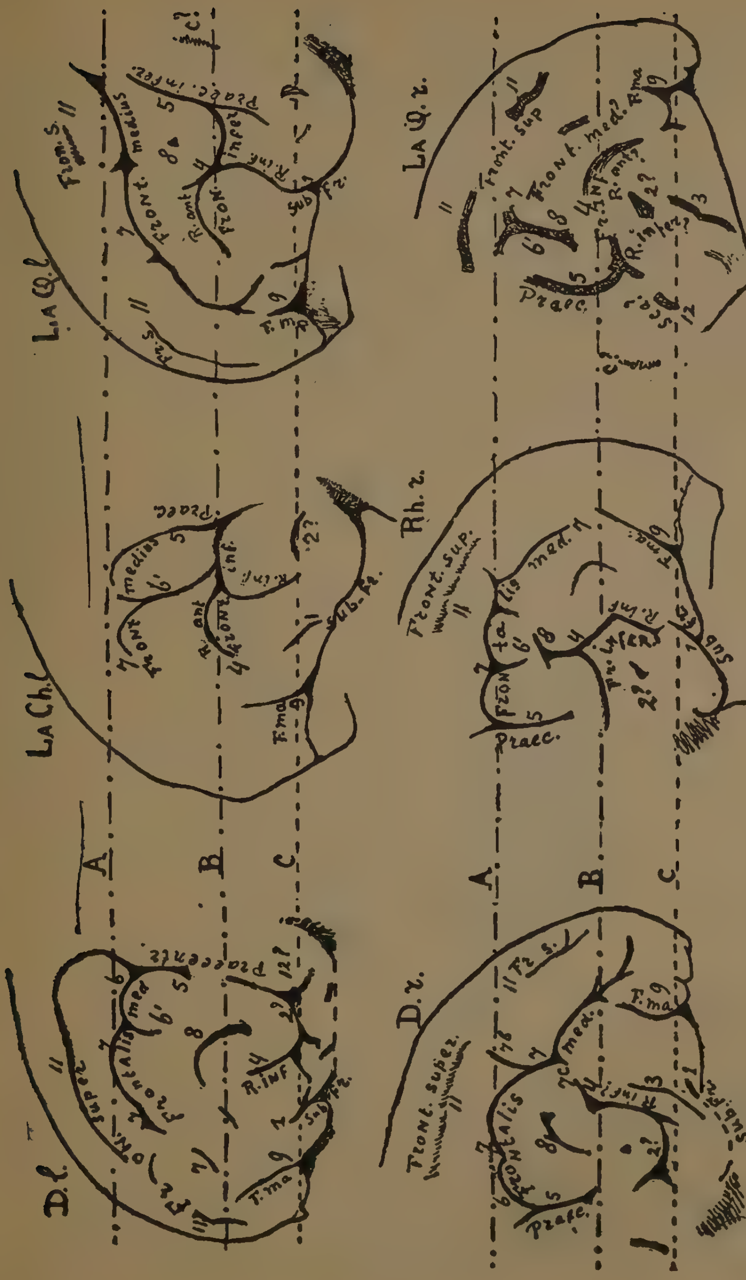

Fig. 2. Fissures on the frontal lobes of the Neanderthal man from Düsseldorf (D.), La Chapelle aux Saints (La Ch.), La Quina (La Q.) and Rhodesia (Rh.); l = left bemisphere, r = right hemisphere.

less distinct than in the ape-man of Trinil, reminds us in several respects of the latter.

This is particularly evident in the fissures 5, 6 and 7 of the right lobe of Pithecanthropus and the Düsseldorf Neanderthal men. On the left lobe of the Düsseldorf and La Chapelle cast an additional offshoot 6′ is indicated. This also occurs on the right side of the Rhodesian cast, a relation reminding us of that found in recent men (fig. 4 A and B). Also the relations at the ventral margin of the lobes — clearly indicated in some Neanderthal men — show a great resemblance with Pithecanthropus, specially as far as regards fissure 1 and 9. Fig. 3 shows the basal course of fissure 1, which I called *S. subfrontalis* [1]). The system of fissures numbered 4, however, varies a good deal in Neanderthal men, and differs from the course of this system in Pithecanthropus. On the left lobe of the La Chapelle and La Quina man it especially shows a larger and frontally higher curve than on the Trinil cast by the nearly horizontal course of its ramus anterior (R. ant. fig. 2).

Fig. 3. Basal aspect of the endocranial cast of the skull of Rhodesia.

I consider the enlargement of the curve of the *S. frontalis inferior* (for this is 4) as an indication of a further development of BRODMANN's subregio

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of man, 7th Ed., Vol. II (p. 618 fig. 223), where also a profile drawing of the original Piltdown cast is given (p. 616, fig. 221), together with a very instructive picture of the endocranial cast of an Australian aboriginal (fig. 222).

[1]) Not to be confused with EBERSTALLER's s. subfrontalis (= calloso-marginalis Autorum).

frontalis inferior, the region that lies underneath the inferior frontal sulcus and that caudally includes BROCA's speech centre (on the left side).

Although it is remarkable that in the Neanderthal casts [1]) only faint impressions of rami anteriores fossae Sylvii are found (fig. 2 : 2 ?) we must remember that SYMINGTON [2]) never found impressions of these branches in his endocranial casts of recent men.

For comparison with Homo recens I use MAZZOTTI's cast which is, just as the other casts, available to everybody, so that everyone may verify my interpretation. *In doing so one has to remember that this cast of Homo recens has been made from the brain itself, while the others are endocranial casts. This means that many more fissures are visible in the former.*

Also in this model we find that groove 1, which, as comparisons with many recent brains show, may be either an independent groove, which may also here be called *s. subfrontalis* [3]), or a branch of the orbitalis externa (the lateral limb of BROCA's "scissure en H"), or it may even be entirely confluent with that limb. Also here, this groove runs backwards underneath the orbital operculum, and may appear again on the surface behind it. The orbital operculum is dorsally bordered by a fissure *2h*, that runs more horizontally than fissure 2 in Pithecanthropus, and, apparently, is the *ramus anterior horizontalis fossae Sylvii* separating the frontal from the orbital operculum.

Groove *2h* — together with the more caudal and perpendicular groove *2a* (*ram. anterior ascendens fossae Sylvii*) embraces the frontal operculum or cape of BROCA — well developed on both sides of this model.

The great development of the frontal operculum also appears from its axial fissure (*fiss. axialis operculi frontalis :* ax. o. fr., fig. 4) that enlarges its surface (not to be confused with the axial furrow of the orbital operculum). The axial furrow of the orbital operculum 3 is pushed somewhat downward by the development of the frontal operculum, and on the left hemisphere is elongated in a frontal direction correspondingly with the enlargment of the frontal curve of the *frontalis inferior* (4), with which it is connected on both sides as in Pithecanthropus.

On the right side the *inferior frontal fissure* (4), in addition to the fissura axialis operculi frontalis has still another caudo-ventral branch, below the former, that also enlarges the surface of the frontal operculum.

The frontal curve of 4 is very large, especially on the left hemisphere (fig. *4B*). From its posterior curve a sulcus proceeds between the *ram.*

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1) In their studies on the casts of La Chapelle aux Saints and La Quina (l. c. infra) BOULE and ANTHONY, sometimes call my fissure 1 : s. p. a. (scissure présylvienne antérieure) or o. (orbitaire) or o. i. (orbitaire interne). They, however, also pointed out that its homology with an orbital sulcus is more probable than with a ramus anterior horizontalis fossae Sylvii.

2) SYMINGTON Sir John Struthers lecture. Edinburgh Medical Journ. Febr. 1915, p. 17. See, however, also BOULE and ANTHONY Neopallial morphololy based on endocranial casts. Journ. of Anat. and Phys. Vol. 51, 1917.

3) Not to be confused with EBERSTALLER's subfrontalis (=calloso-marginalis of the author's).

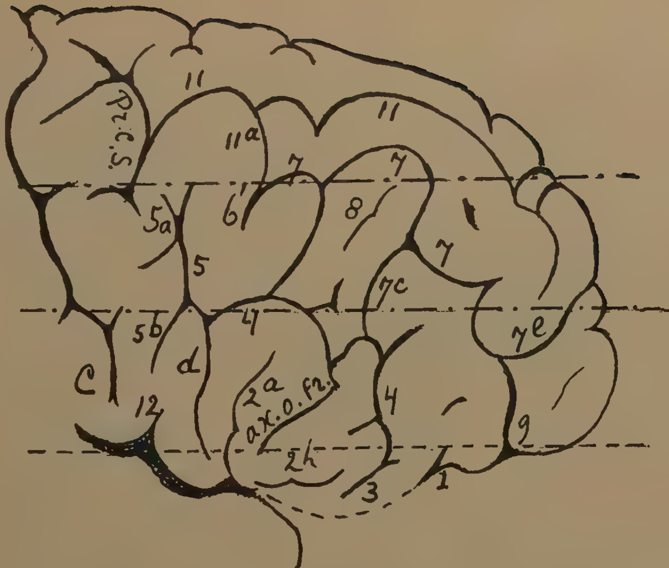

Fig. 4A. Fissures on the right frontal lobe of a recent man.

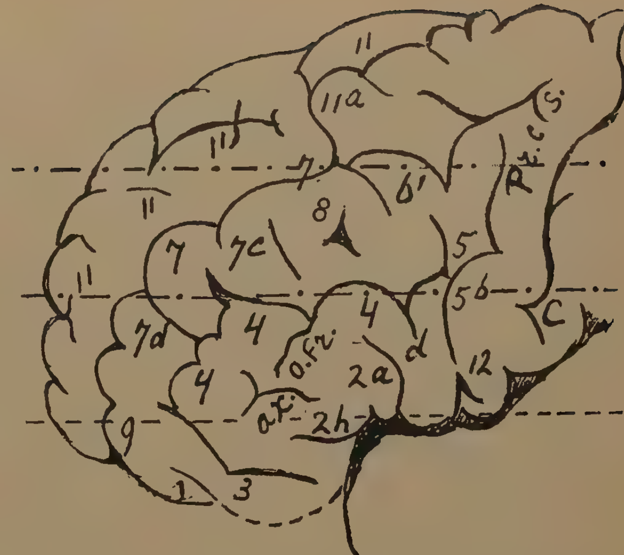

Fig. 4B. Fissures on the left frontal lobe of a recent man.

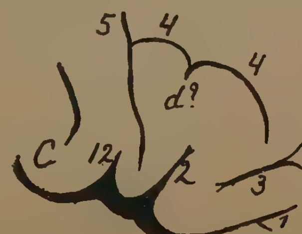

Fig. 4C. One of Retzius' men in whom only one ramus anterior fossae Sylvii (2) occurs (cf. the right lobe in fig. 1).

anterior ascendens fossae Sylvii and the *f. precentralis* (5) the *f. diagonalis* (*d*) of which no trace is visible in Neanderthal casts. This, however, does not prove that a diagonalis was missing in Neanderthal brains since SYMINGTON (l.c.) showed that this part of the pallium (in recent men at least) gives no or only very faint impressions on the skull.

Here the question of the homology of fissure 2 in Pithecanthropus must be discussed, and, with it, the question of the homology of fissure 12 in the ape-man of Java, and 12 ? in Neanderthal casts.

At first sight one might be inclined to homologize fissure 12 of the Trinil cast with 2*a* of recent man, and 2 of the Trinil cast with 2*h* of the latter. DUBOIS seems to have done this in his lecture at Cambridge (l.c. primo, p. 83) saying : "the two segments of this figure (he apparently refers to the curves of the frontalis inferior : 4) encircle the limbs of perfectly definite Y shaped anterior branches of the fissura Sylvii, the stem of which is about 1 cm long".

In his last description, however, (l.c. secundo, p. 273) he speaks only of "a strong front branch of the Sylvian fissure".

I am also inclined myself to accept the presence of only one single anterior branch of the Sylvian fossa in Pithecanthropus for the following reasons. The *ramus ascendens anterior f. S.* always lies in front of the *precentral fissure* (5) while in Pithecanthropus fissure 12 lies behind the ideal elongation of the precentral. So fissure 12 of Pithecanthropus can be only the *f. subcentralis anterior,* a fissure very constant also in anthropoids. Consequently fiss. 2 in the Trinil cast is a single ram. anterior f. S., as sometimes also occurs in anthropoids, where two separate rami anteriores f. S. never occur according to EBERSTALLER [1]), though according to my experience and that of others a *Y* shaped ramification is occasionally observed here (cf. fig. 5 left lobe). Even in man a single ramus anterior is no exception. EBERSTALLER found a single ram. anterior in 24 % of his German brains, CUNNINGHAM [2]) in 27 % of his Irish material, and QUANJER [3]) in 18 % of his Dutch material. It is remarkable that CUNNINGHAM as well as QUANJER found this condition three times oftener on the right than on the left hemisphere. So CUNNINGHAM saw a single ramus anterior f. S. in 41 % of his right hemispheres. Also RETZIUS [4]) found this condition quite frequently in Swedish brains. In fig. 4C I reproduce one of his cases (l.c. plate 67, fig. 1). The resemblance of this case with the relations in Pithecanthropus (fig. 1 right lobe) is striking and not only concerns fissure 2, but also the combination of 12 and 2 and

1) EBERSTALLER. Das Stirnhirn. Urban und Schwartzenberg, Leipzig und Wien, 1890.

2) CUNNINGHAM. Contribution to the surface anatomy of the cerebral hemispheres with a chapter on cranio-cerebral topography. Memoirs N⁰. 7 Roy. Irish Academy of Sciences 1892 and CUNNINGHAM. The insular district in the cerebrum of anthropoid apes. Journ. of Anat. and Phys., vol. **31**, 1897.

3) QUANJER. Zur Morphologie der Insula Reilii und ihre Beziehung zu den Opercula. Petrus Camper, Deel 2. 1902.

4) RETZIUS, Das Menschenhirn, Stockholm, 1896.

the surrounding *f. frontalis inferior* (4) and its ventral offshoot 4′ (= *d*?, a rudimentary diagonalis?). Similar relations may be found in human fetuses.

Considering the fact that on both hemispheres of the Pithecanthropus only a single anterior branch of the f. Sylvii is indicated and that on the left hemisphere the inferior frontal convolution is still smaller than on the right, we have no morphological reasons to assume that this creature possessed the ability of speech although neither can it be proved that it could not speak [1]). In recent men the inferior frontal sulcus very often has dorsal branches, also indicated in the Neanderthal (see fig. 2 La Chapelle and La Quina l.; Düsseldorf and Rhodesia r.) but not in the Trinil cast.

The *intermediate fosset* 8, occurring in the ape-man and Rhodesian man on the right, in the Düsseldorf on both and in the La Quina man on the left hemisphere, is also observed in recent men in the form of a large triradiate star situated somewhat more frontally (see specially the left side of the model; on the right lobe this fosset seems to be grown out in the form of a transvere fissuret 8.

The *inferior precentral fissure* (5) in recent man frequently has also a ventro-caudal branch *5b*, which, however generally extends far downward as does the diagonalis. This ventral extension is absent in the Trinil cast and probably did not occur on the Pithecanthropus brain, as (at least on the right) the adjacent fiss. 2 is so well expressed.

In my Neanderthal casts a ventral extension of the inferior precentral is oftener indicated on the left hemispheres than on the right which does not however, prove that it did not exist on the right lobes, since SYMINGTON has observed that this region is not very apt to make impressions (see above).

The downward curve 6′ of the midfrontal sulcus (7) present in figs 4*A* and 4*B* is also indicated in several Neanderthal men. The midfrontal sulcus (7) itself in fig. 4 continues in the superior precentral, which in the Neanderthal casts it only does in the La Quina man [2]). In all others it continues in the inferior precentral.

Frontally, where the midfrontal fissure curves down, it either shows a tendency to connect or a real connection with the *fronto-marginal* (9) in recent men, as is also indicated in Neanderthalmen and Pithecanthropus.

Dorsally another branch (7*e*), also indicated in the Trinil and Düsseldorf casts, runs to the medial margin of the brain in fig. 4A.

Of the *superior frontal sulcus* the caudal part was absent in the Neanderthal and Trinil casts, but it has certainly been present seeing that it constantly occurs in anthropoids. The frontal end of 11 in fig. 4B reminds us strongly of the relations in the Neanderthal casts (fig. 2 D.r.) by its discontinuity.

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[1]) According to many linguists human speech originates in uttering emotional and imitative sounds. As these faculties also occur in several animals, it seems very difficult to say where „speech" begins.

[2]) And perhaps in the Gibraltar cast (see KEITH, Vol. II, fig. 223).

We thus find a transition between the Pithecanthropus, Neanderthal and recent men, but also marked differences, e.g. in the development of the frontal operculum, not delimited in the Trinil cast and not clearly delimited in Neanderthal casts while in Homo recens in 86 % of the left and in 59 % of the right hemispheres it is well limited by two anterior branches of the fossa Sylvii. Furthermore, the curve of the inferior frontal sulcus is considerably enlarged in recent men (especially on the left lobe) a process already indicated in Neanderthal casts. There also is a marked increase of the area below the mid frontal sulcus, which area in Neanderthal but specially in recent men is larger than on the Trinil cast. In recent men a special enlargement may be observed in the region between the fosset 8 and the precentral sulcus (the so called foot of the midfrontal convolution).

As far as concerns the increase of the frontal operculum we know that this operculum, and the area immediately in front of it, corresponds to BRODMANN's subregio frontalis inferior that acts a large part in speech (on the left in right handed people). The functions of the cortex between 8 and 5 are less known. This area belongs to the regio frontalis granularis of BRODMANN [1]) of which this author also showed the progressive development in primates and in men. According to SAHLI's [2]) researches the foot of the midfrontal convolution contains *Prévost's* centre for the conjugated deviation of head and eyes. Although this frontal center already occurs in carnivora, monkeys and apes (FERRIER) [3]), it may be that its surrounding is specially concerned in the enlargement of the foot of the mid frontal convolution in recent men. This would not be so strange since the conjugated deviation of the eyes and head is an important function in circum-spection, enabling men (and animals) to fix lateral objects, for which reason it has been also called "spy centre".

Before ending I shall compare the frontal lobes of the Trinil cast with those of a Chimpanzee, the fissures of which show a great resemblance with those of the former. It is interesting that also SCHWALBE [4]) and WEINERT [5]) considered the Chimpanzee as the anthropoid whose skull structure comes nearest to that of the Trinil man, while MINGAZZINI [6]) found the Chimpanzee's fissuration coming nearer that of men than the Orang's fissuration does. Similarly KEITH and MOLLISON believe the

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[1]) BRODMANN. Vergleichende Lokalisationslehre der Groszhirnrinde in ihren Prinzipien dargestellt auf Grund des Zellenbaues. Joh. Ambr. Barth, Leipzig 1909.

[2]) SAHLI. Beitrag zur corticalen Lokalisationslehre des Centrums für die conjugierte Seitwartsbewegung etc. Deutsches Archiv für Klin. Medizin, Bnd. 86.

[3]) FERRIER, Vorlesungen über Hirnlokalisation, p. 32, Deuticke, Leipzig u. Wien, 1892.

[4]) SCHWALBE. Studien über Pithecanthropus erectus DUBOIS. Zeitschr. f. Morphologie und Anthropologie, Bnd. 1, 1899.

[5]) WEINERT. Pithecanthropus erectus. Zeitschr. f. Anatomie und Entwicklungsgesch., Bnd. 87, Heft 3 und 4, 1928.

[6]) MINGAZZINI. Beiträge zur Morphologie der äuszeren Groshirnhemisphärenoberfläche bei den Anthropoïden (Schimpanse und Orang). Arch. f. Psychiatrie, Bnd, 85, 1928 (p. 212).

Chimpanzee to be nearer related to man than the Orang and even the Gorilla.

If in fig. 5 one eliminates the rostrum orbitale, which is largely missing in the Pithecanthropus cast, its resemblance to that of the Trinil man becomes still greater. But even then the more compressed shape of the Chimpanzee 's brain is striking, the shortness of the frontal lobes as compared with their height.

The antero-posterior shortening of the lobes, already expressed in the brachencephaly of this animal (index 84.5), is equally revealed in the course of their sulci which appear to be more pushed backward than in the Trinil man.

This is already seen from the course of the *S. fronto-orbitalis* [1]) (1), which fissure is homologous to the *f. subfrontalis* mihi in Pithecanthropus, Neanderthal and recent men, but the dorsal part of which runs much steeper in the Chimpanzee and even shows a backward inclination on the right hemisphere of the latter [2]).

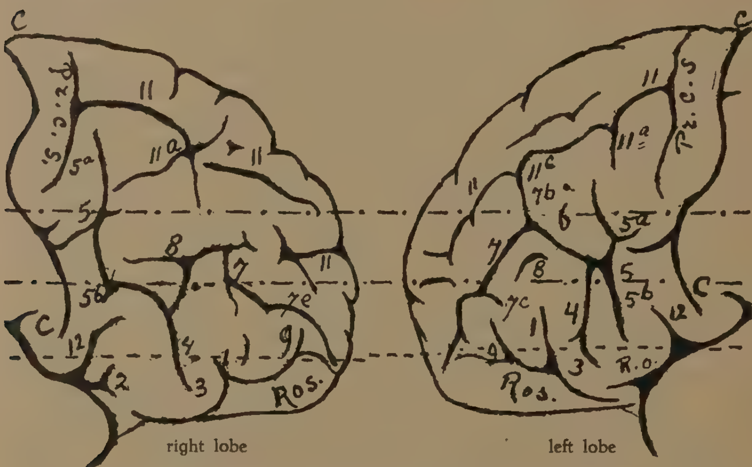

Fig. 5. Fissures on the frontal lobes of Troglodytes niger.

[1]) For other human homologies of the fronto-orbital sulcus mentioned by former authors I refer to my "Vergleichende Anatomie des Zentralnervensystems der Wirbeltiere und des Menschen", Vol II, p. 1147. I want to emphasize only that ELL. SMITH was right when he supposed that the larger part of the dorsal section of this sulcus disappears from the convexity in men. This process already begins in the Chimpanzee (c.f. the right and left lobe in fig. 5) in comparison with the Orang-Outan. Concerning the ventral (orbital) part of this fissure I can no more share the opinion of those authors who supposed that this part is represented by a the S. limitans anterior insulae in men. This part returns as s. subfrontalis, that may be very small in recent men or be connected with the fiss. orbitalis externa. After writing this I found that also KEITH has established the same homology of the fronto-orbital in his report on the Galilee skull (l.c. supra).

[2]) The steeper course of the dorsal part of the fronto-orbitalis is still more striking in the Orang-Outan whose brachencephaly is also greater (87.7).

With the *inferior frontal sulcus* (4) the more caudal position is equally evident, its curve in the Chimpanzee being much steeper, specially on the left hemisphere than in Pithecanthropus. The *midfrontal sulcus* (7) of the left lobe (fig. 5) resembles very much the course of the same sulcus on the right lobe of the Trinil cast, although its curve again is slightly more compressed.

In both a connection with the superior frontal sulcus occurs by means of 7*b* and 11*c*. On the right hemisphere the midfrontal sulcus is hardly recognized and apparently connected with the intermediate fosset 8, very large in fig. 5 and connected with the inferior frontal fissure (cf. the Rhodesia Neanderthalmen), as well as with the mid frontal sulcus (cf. the the right lobe of La Quina cast).

A point of resemblance between the right hemisphere of the Chimpanzee with the same in Pithecanthropus is the presence of a single ram. anterior f. Sylvii (2), which, however, in the Chimpanzee shows a bifurcation (thus establishing the well known *Y* form of this sulcus, which may also occur in men). This fissure (2) ends caudally in the fossa Sylvii in which also the subcentralis anterior (12) disappears.

It is evident that the area situated underneath the inferior frontal sulcus in the Chimpanzee is still smaller than in the Trinil cast, which not only appears from the steeper curve of 4, but also from the distance between 1 and 12 being smaller in the latter. Although this difference partly results from the smaller size of the orbital operculum it is largely due to the small development of the area situated behind and on top of it, the operculum frontale, which in man acts such a large part in speech.

In general lines, however, the resemblance between the Chimpanzee and Pithecanthropus is very great and the fissural pattern of the latter apparently forms an intermediate condition between that of the Chimpanzee and of Neanderthal men.

In its general shape the Trinil cast, however, comes nearer that of the brain of Hylobates syndactylus (encephalic index 80), especially as far as concerns the sagittal height indices of the calotte part. It also struck DUBOIS that the norma lateralis of his cast resembles in many respects that of Hylobates syndactylus. Nevertheless by its extremely simple fissuration, Hylobates is further removed from Pithecanthropus than any other anthropoid is.

Considering also its large skull capacity and cephalization coefficient [1]) it does not seem impossible that Pithecanthropus was a Hominide.

Its brainweight is generally compared with that of the average European (1300 gr. of men and women). If, however, we compare it with the lower living representatives of mankind, it approaches human conditions still more. So the brain weight in Australian aboriginals, according to DAVIS [2]),

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[1]) DUBOIS calculated the cephalization coefficient of Pithecanthropus to be about twice as large as that of anthropoid apes, taking the femur as an indicator for body size and weight.

[2]) DAVIS. Contributions towards determining the weight of the brain in different races of men. Phil. Transact. of the Royal Soc. London. Vol. 158, 1869.

whose capacity estimations well accord with those found by TURNER (1230 cc) [1]) and DUCKWORTH [2]) (1246.5 cc), is no more than 1173 gr. (average for 17 men and 7 women).

DUBOIS' first estimation of the capacity of the Trinil skull was 855, which he later raised to 900 cc.

MC. GREGOR [3]) calculated the endocranial capacity to be about 940 ccm. WEINERT (l.c.) who took the average of four different methods of calculation found a still higher capacity, viz. 1000 ccm. Considering the fact that dried skulls always have a smaller capacity than fresh ones and adding 30 cc for this (50 cc in man) the original capacity might have been as much as 1030 ccm. Accepting as brain volume 91 % of this capacity we get 937 ccm brain, which, with an average specific weight (for white and grey matter) of 1.037 would give a brainweight of 972 grams, whereas the heaviest anthropoid brain hitherto weighed did not exceed 440 grams [4]).

So the brain of the ape man of Java weighed only 200 grams less than of Australians. Similar comparisons are made by DUBOIS with Andamanese and Weddahs (l.c. primo p. 85).

In connection with the intermediate character of Pithecanthropus I also may mention ELL. SMITH's *lunate sulcus* the position of which on the right corresponds with the top of the lambda suture, while on the left it seems to lie much more caudally. DUBOIS emphasized that this sulcus in all anthropoids lies before the lambda suture, while it lies behind it, in men. In recent man this sulcus is more frequently expressed on the left (ELL. SMITH).

The same holds good for the lunate sulcus in Neanderthalmen, where according to BOULE and ANTHONY [5]) it is visible behind the lambda suture in the left hemisphere of the La Chapelle cast. ANTHONY [6]) indicates it in a similar position in the *La Quina* cast. I found it more clearly and in a similar position on the left hemisphere of the Düsseldorf cast, where even the posterior calcarine and part of the superior occipital fissure may be indicated.

ELL. SMITH [7]) mentions seeing this fissure on both sides in the Rhodesia cast, just in front of the lambda suture, but I consider the symmetrical

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[1]) TURNER. Report of the Challenger. Zoology, Vol. **10**, part. 1 crania, London 1887, quoted from BURKITT and HUNTER. Description of a Neanderthaloid australian skull etc. Journal of Anatomy and Physiol. Vol. **57**, 1922.

[2]) DUCKWORTH. Studies in Anthropology, Cambridge University Press, 1909,

[3]) MC. GREGOR, Recent studies on the skull and brain of Pithecanthropus. Nat. History, Vol. **25**, p. 555, 1925.

[4]) This refers to FICK's Outan brain mentioned by ZIEHEN in Bardelebens Handbuch der Anatomie p. 365, but HAGEDOORN found a maximum skull capacity in one of BOLK's Gorillas of 655 ccm. See Anat. Anz. Bnd. **60**, 1925—26, p. 417.

[5]) BOULE et ANTHONY. l'Encéphale de l'homme fossile de la Chapelle aux Saints. l'Anthropologie, Vol. **22**, 1911. See also Journ. of Anat. and Phys, Vol. **51**, 1917.

[6]) ANTHONY. l'Encéphale de l'homme fossile de La Quina. Bulletins et mémoires de la Société d'Anthropologie de Paris, Mars. 1913 (fig. 10, p. 159).

[7]) ELL. SMITH. Rhodesian man and associated remains. Brit. Museum of Nat. Hist. publ. 1928.

C. U. ARIËNS KAPPERS The fissures on the frontal lobes of Pithecanthropus erectus Dubois compared with those of Neanderthal men, Homo recens and Chimpanzee.

Fig. 1
right hemi-
sphere.

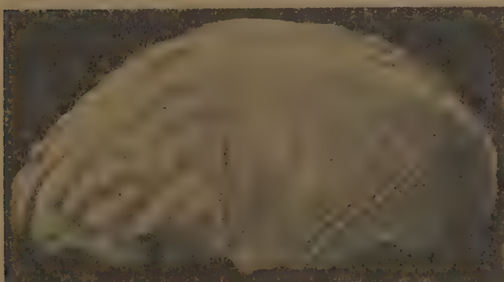

Fig. 2
left hemi-
sphere.

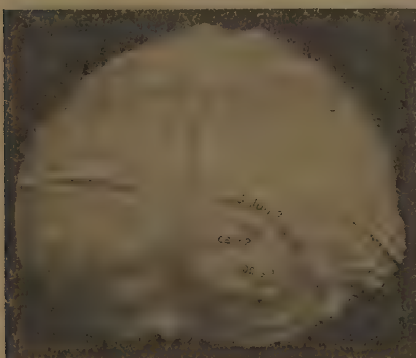

Fig. 3
occipital
lobes

Vlassopoulos' drawings of the endo-cranial cast of Pithecanthropus erectus Dubois. In fig. 2 behind and under 9 (which refers to the sulcus in front of it) a vestige of fiss. 1 is seen.

whose capacity estimations well accord with those found by TURNER (1230 cc) [1]) and DUCKWORTH [2]) (1246.5 cc), is no more than 1173 gr. (average for 17 men and 7 women).

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2) DUCKWORTH. Studies in Anthropology. Cambridge University Press, 1909.

3) MC. GREGOR. Recent studies on the skull and brain of Pithecanthropus. Nat. History, Vol. 25, p. 555, 1925.

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5) BOULE et ANTHONY. l'Encéphale de l'homme fossile de la Chapelle aux Saints. l'Anthropologie Vol. 22, 1911. See also Journ. of Anat. and Phys. Vol. 51, 1917.

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7) ELL. SMITH. Rhodesian man and associated remains. Brit. Museum of Nat. Hist. publ. 1928.

C. U. ARIËNS KAPPERS: THE FISSURES ON THE FRONTAL LOBES OF PITHECANTHROPUS ERECTUS DUBOIS COMPARED WITH THOSE OF NEANDERTHAL MEN, HOMO RECENS AND CHIMPANZEE.

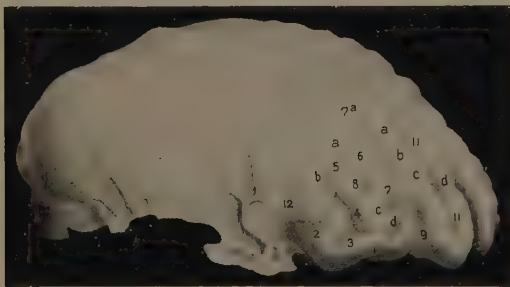

Fig. 1 right hemisphere.

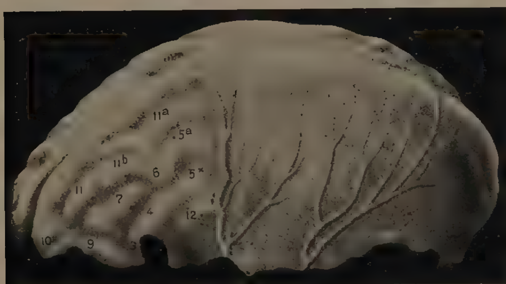

Fig. 2 left hemisphere.

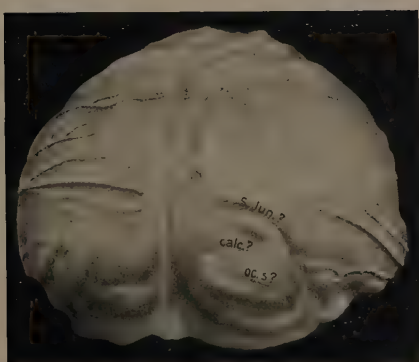

Fig. 3 occipital lobes

Vlassopoulos' drawings of the endo-cranial cast of Pithecanthropus erectus Dubois. In fig. 2 behind and under 9 (which refers to the sulcus in front of it) a vestige of fiss. 1 is seen.

indentation on these places as a result of a thickening of the posterior border of the parietal bone, which I also encountered in different recent skulls.

Concerning the *central sulcus* in Pithecanthropus and Neanderthal men I have to limit myself to suppositions concerning its ventral part. Perhaps a shallow groove behind 5*b* on the left and behind 5* on the right side of the Trinil cast, just in front of the anterior branch of the arteria meningea media, represents this part. If this is true — of which I am not sure — the central end of the Rolandic fissure would closely approach the *s. subcentralis anterior*, and thus show a higher condition of development than in anthropoid apes, where its ventral end usually curves backward (fig. 5), although the limit of the sensory and motor area already in the Chimpanzee (BRODMANN [1]) takes a fronto-ventral course (an example of the retardation of sulci in adapting themselves to cytotectonic bordering lines [2]). For Neanderthal men ANTONY supposes the ventral end of this fissure also to lie near the anterior branch of the arteria meningea media (l.c. primo, p. 165 and l.c. secundo, p. 164 ; cf. also C? in my fig. 2 of La Quina r. and l.).

The length-width index of the endocranial cast of Pithecanthropus (81.2 according to my calculation) does not differ so much from that of Neanderthal men. According to ANTHONY's calculation this index in the Gibraltar cast is even 81.5 (according to mine 79.5). The latter found 78.3 and 78.6 for the Düsseldorf and La Chapelle casts, while I found the Düsseldorf cast to have even a little more (79.6).

From this it appears that also in the length width index of its cast (81.2) Pithecanthropus comes very near Neanderthal men, nearer than to the Chimpanzee (84.5) [3]).

In a following paper I shall compare the endocranial casts of various Neanderthal men, and publish the drawings of these casts, made by the scientific artist Mr. CHR. VLASSOPOULOS, to whom I am also indebted for the excellent plate joining this paper.

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1) BRODMANN. Neue Ergebnisse über die vergleichende histologische Lokalisation der Groszhirnrinde mit besonderer Berücksichtigung des Stirnhirns. Verh. An. Ges. 1912.

2) For other examples of this retardation of sulci, see KAPPERS. Cerebral localization and the significance of sulci. Report of the XVII[th] international Congress of medicine, London, 1913.

3) For brain indices see "The influence of cephalization coefficient and body size on the on the form of the forebrain in mammals. Proceed. of the Kon. Akad. v. Wetensch., Amsterdam, Vol. 31, 1927.

Anatomy. — *Further communication on the fissures of the frontal lobes in Neanderthal men.* By C. U. ARIËNS KAPPERS.

(Communicated at the meeting of January 26, 1929).

In the meeting of December 22nd 1928 I gave a description of the fissures on the frontal lobes of Pithecanthropus erectus Dubois, comparing them with those of the Chimpanzee, Neanderthal- and recent men. In the present communication I shall compare the frontal lobes of different Neanderthal men with one another, publishing at the same time the drawings, made by the scientific artist Mr. CHR. VLASSOPOULOS, of some Neanderthal casts.

I shall discuss the s. frontalis superior, medius and inferior, their relation to the precentral fissures, the intermediate fosset (8), and add a few remarks concerning the inferior marginal part of the frontal lobe.

The *s. frontalis superior* [1]) gives very few impressions in Neanderthal-casts, especially on the caudal part of the frontal lobe. This fits in very well with SCHWALBE's [2]) and SYMINGTON's [3]) experiences with the casts of recent men, from which appeared that the impressions of the convolutions and the juga caused by grooves are less pronounced proceeding from the bottom of the frontal lobe to its dorso-caudal surface.

On the La Chapelle cast there is even no indication at all of a superior frontal fissure. On three casts, however, those from Düsseldorf (left and right), La Quina (left) and Rhodesia, a large part of this fissure is indicated.

In anthropoids the frontalis superior may or may not be connected with the superior precentral fissure (see fig. 5 of my first communication). It may moreover be interrupted in its own course. The same is observed in adult Europeans. Which of these conditions prevails in Neanderthal men is difficult to say on account of the lack of impressions.

Only on the Düsseldorf Neanderthal [4]) and the Podbaba man (left

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[1]) Of a sulc. fronto-mesialis (CUNNINGHAM), so often occurring in recent men, and occasionally in anthropoids (separate dimples medially to the superior frontal fissure) no races are observed on the casts. Its very dorsal position apparently prevents this.

[2]) SCHWALBE. Ueber die Beziehungen zwischen Innenform und Auszenform des Schädels. Deutsches Archiv f. Klin. Medizin, Bnd. **73**, 1904.

[3]) SYMINGTON. On the relations of the inner surface of the cranium to the cranial aspect of the brain. Sir JOHN STRUTHERS lecture. Edinburgh medical Journ. 1915 and Endocranial casts and brainform: a criticism of some recent speculations. Journ. of Anat. and Phys., Vol. **50**, 1916. (See, however, also BOULE and ANTHONY, Ibid. Vol. 51, 1917).

[4]) On the left hemisphere the supposed connecting fissure of the frontalis superior with the precentral is only indicated by its lateral wall. i. l. by the mid-frontal convolution. The impression of its medial wall, the superior convolution is flattened. — In the Podbaba cast (left) the latter convolution seems to be indicated.

hemispheres) a connection of the frontalis superior with the precentral is slightly indicated.

It is, however, quite possible that the superior frontal fissure extended caudally beyond this point as it may do in recent men.

On the other hand, the second characteristic of the frontalis superior — the interruption in its course — is clearly present in some casts.

Even in Pithecanthropus (right hemisphere) this fissure becomes less deep about the level where it is approached by 7e (cf. fig. 1, first communication).

Although its more shallow character at that level is evident, I have no reason to accept that a real interruption occurred here, the less so as the brain fissures were of course deeper than those on the cast.

It is, however, interesting that at the same level there is a distinct interruption of this endocranial groove in the Düsseldorf man (left and right). The same seems to occur on both sides of the La Quina cast. Although this interruption may also occur in anthropoids (see fig. 5 of my first communication and EBERSTALLER's [1]) figures 7 and 8), it is especially frequent in recent man. CUNNINGHAM [2]) even distinguished two parts in the superior frontal fissure, a frontal accessory part and a main hind part, which often originate separately in ontogenesis. Evidently the same distinction can be made in Neanderthal men.

A third point in its morphology are the connections of this fissure with the s. frontalis medius : the rami connectentes, which may proceed as well from the main part of the frontalis superior as from its frontal accessory part (about the level of 7*b*).

CUNNINGHAM observed the latter in 44 % of his Irish brains (l.c. p. 69). Also these connections are not specifically human, they are observed as well in anthropoids as in DUBOIS' ape man. That in the Neanderthal-casts the indications of such connections are only rare, is not strange since the impressions at this level are generally vague, and these connections moreover are less deep than the superior frontal itself. Traces of it are observed on the right hemisphere of the Düsseldorf man (at 7*b*) and at the same spot in the Rhodesian by a dorsal branch of 7. It is, however, difficult to say whether these branches reached the superior frontal.

The impressions of the *sulc. frontalis medius* are much more evident than those of the former. Still this groove is realized in recent men only much later than the superior and inferior frontal fissure (by EBERSTALLER in 1890, p. 72). In his description this author says that this groove "in seiner typischen Gestalt" does not join the precentral but only begins

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[1]) EBERSTALLER. Das Stirnhirn. Ein Beitrag zur Anatomie der Oberfläche des Groszhirns. Urban u. Schwarzenberg Wien u. Leipzig, 1890.

[2]) CUNNINGHAM. Contributions to the surface anatomy of the cerebral hemispheres with a chapter on cranio-cerebral topography. Memoires N⁰. VII, Roy. Irish Acad. Dublin 1892.

(usually by a transverse piece) quite a distance in front of it. It is different in Neanderthal men. In all my casts its connection with the precentral (mostly the inferior) seems to exist. On the La Quina left hemisphere it connects with the superior precentral, a condition that may also occur in KEITH' Gibraltar cast and in recent men.

Concerning anthropoids I must confess that this fissure is very often described (recently again by MINGAZZINI [1]) as S. frontalis inferior. This is caused by the fact that the frontalis inferior in anthropoids may fail, or — on account of its steep course in those animals (see fig. 5 of my first contribution) — is not recognized as such, and their sulc. medius, often connected with the precentral reminds one of the frontalis inferior of men.

EBERSTALLER came nearer the truth when he compared the sulc. rectus of monkeys and apes with the frontalis medius of men, although I am more inclined to consider the rectus as a forerunner of the fronto-marginal fissure, which, however, is very frequently continuous with the frontalis medius (cf. also KEITH: Report on the Galilee skull, 1927, p. 103).

In the Trinil cast where all frontal sulci are clear in their arrangement and form a splendid base of interpretation, the frontalis medius (7) is also continuous with the inferior precentral (5) and can be also distinguished from the fronto-marginal (9).

In the Neanderthal men the medius has the following characteristics: Everywhere where the region in which it lies is well expressed. It is continuous mostly with the inferior precentral or with the superior [2]) precentral. The latter is indicated on the left hemisphere of the La Quina cast, where consequently the mid-frontal sulcus runs ventro-frontally in its initial course. In all other casts the frontalis medius (7) in its initial course shows a dorsal curve (6), immediately in front of the inferior precentral. This curve may continue with a ventral branch (6′) which is hardly (or not) indicated on the right hemisphere of the Düsseldorf Neanderthal men, that thus resembles the relation in the ape-man (fig. 1, right hemisphere of my first contribution), and the Chimpanzee (fig. 5, left hemisphere, ibidem) very strikingly.

The ventral branch 6′ is, however, well pronounced on the left hemisphere of the Düsseldorf and La Chapelle Neanderthal man and on the right side of the Rhodesian and probably of the La Quina cast.

A similar branch (6′) is very frequent in recent men. It even occurs on EBERSTALLER's scheme where it is represented, not nominated, as a triradiate sulcus between his F. 2, and the top of his p.c.i. But here, as often in recent man, the connection of 6 with the inferior precentral is inter-

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[1]) MINGAZZINI. Beiträge zur Morphologie der äuszeren Groszhirnhemisphärenoberfläche bei den Anthropoiden (Schimpanse und Orang). Archiv f. Psychiatrie und Nerven Krankheiten, Bnd. **85**, 1928.

[2]) This also may occur in recent men. See LANDAU: Ueber die Furchen an der Lateralfläche des Groszhirns bei den Esten. Zeitschr. f. Morph. u. Anthrop. Bnd. **16**, 1913, p. 248.

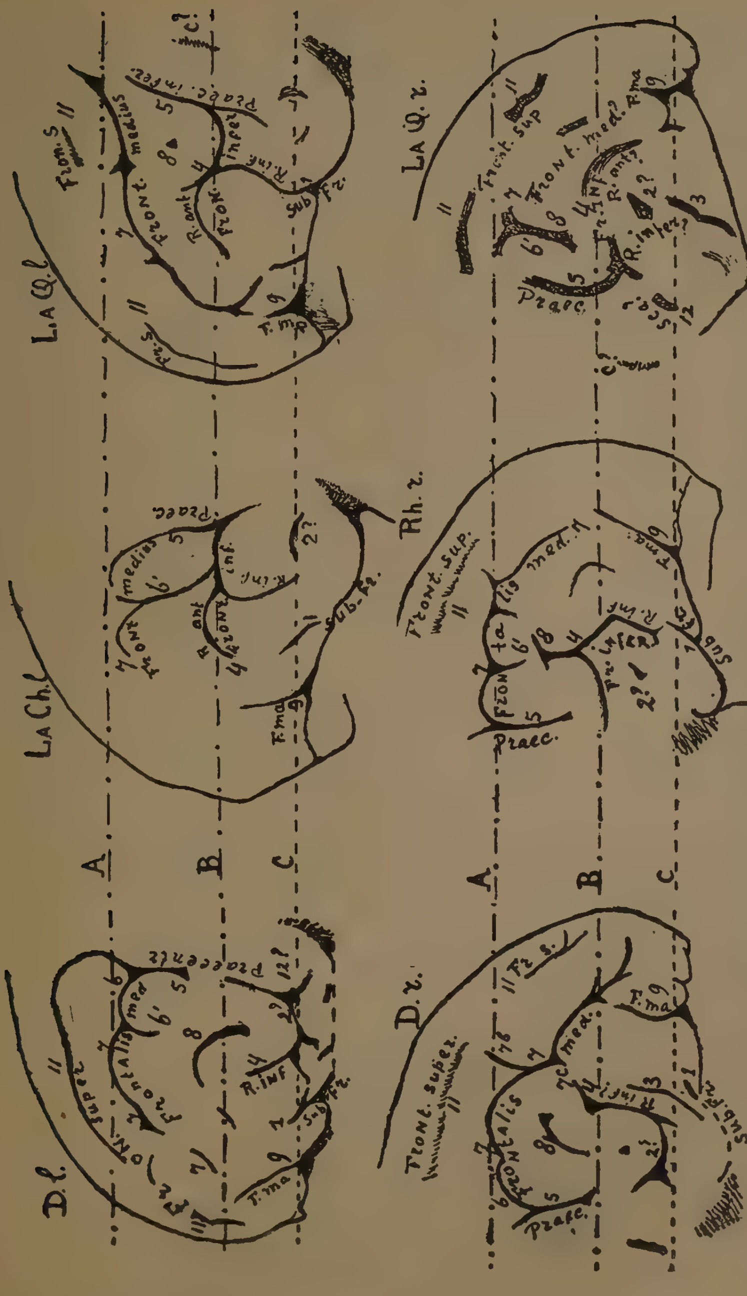

Fig. 1. Fissures on the frontal lobes of the Neanderthal man from Düsseldorf (D.), La Chapelle aux Saints (La Ch.), La Quina (La Q.) and Rhodesia (Rh.) : l = left hemisphere, r = right hemisphere.

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rupted (see also WINKLER [1]) and LANDAU, l.c.). In other cases it is connected with the superior precentral (see fig. 4 of my former contribution).

As, however, the connection of the frontalis medius with the inferior precentral is also frequent in Chimpanzees, and even very pronounced on both sides in the ape man of Trinil, this condition, prevailing in Neanderthal men must be considered as more primitive than in recent men. I am inclined to believe that the interruption of its connection with the inferior precentral and its eventual attachment to the superior precentral, so frequently observed in recent men (fig. 4, l.c.), is a result of the considerable development of the convolution underneath it, the foot of the second frontal gyre, the development of which in recent men in comparison with Neanderthal men I already (l.c.p.) emphasized on account of other reasons.

The medial frontal fissure is further characterized in Neanderthal men by a second curve finishing at 7*c* in the Düsseldorf cast (D.r.). Then it proceeds frontally eventually approaching the interhemispherial cleft with the medial branch of a bifurcation, the lateral branch of which turns in the direction of the fronto-marginal. These relations are similar in Pithecanthropus and in recent men, where the frontalis medius may continue in the fronto-marginal.

Eventual connections between the medial and superior frontal are discussed above.

Also indications or tendencies of connections with the inferior frontal sulcus occur, so in the Düsseldorf man on the right hemisphere at 7*c* as in the Rhodesian between 6′ and 8, in the La Chapelle man at 6′.

It is very remarkable that the mid-frontal sulcus, whose existence in present races is only so recently realized (vide supra) and which is mostly broken into pieces in the Javanese (KOHLBRUGGE [2]), l.c. p. 82), while SERNOFF [3]) could state its occurrence in only 17 % of his Russians, is such a constant and continuous groove in Neanderthal men, where nobody can doubt its existence, and where also ANTHONY [4]) rightly described its occurrence in the La Quina cast, and equally observed its relation to the fronto-marginal fissure. KEITH found this connection even indicated on his Australian cast (Antiquity of Man, 7th Ed., Vol. II, fig. 222).

Before proceeding to the inferior frontal sulcus I have to say a few words concerning the *intermediate fosset* or fissuret 8, lying between the

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1) WINKLER. The relative weight of human circumvolutions. PETRUS CAMPER Deel 1. 1902 Plate 1 fig. 2. (sulc. frontalis intermedius).

2) KOHLBRUGGE. Die Gehirnfurchen der Javanen. Eine vergleichend-anatomische Studie. Verhandl. d. K. Ak. v. W. Amsterdam, 1906 Sectie II: "Ein eigentlicher S. frontalis medius fehlt den Javanen. Im Gyrus frontalis medius seht man statt dessen fast nur transversale Furchen."

3) SERNOFF (russian), quoted from LANDAU l.c. supra.

4) ANTHONY. l'Encéphale de l'homme fossile de La Quina. Bulletins et mémoires de la Soc. d'Anthropologie de Paris, Mars, 1913 (p. 158, fig. 10).

middle and inferior frontal fissure. I have already pointed out that fosset 8 also occurs in the Trinil cast (l.c. right hemisphere fig. 1), and that analogous intermediate fossets may occur in recent men. In the Neanderthal man of Düsseldorf this fosset is enlarged to a fissure that ends in the middle of the missing connection between the frontalis inferior (4) and precentralis inferior (5), in which connection it continues in the Rhodesian. I found an exactly analogous condition in an Australian aboriginal's brain.

On the La Quina cast (right hemisphere) this fosset seems to continue in the mid-frontal sulcus (7) by means of 6′. The relations are, however, not clear enough here to provide full certainty about this. The same configuration is, however, observed by ANTHONY (l.c. p. 162, fig. 11).

Besides already in the Chimpanzee (right hemisphere fig. 5 of my first paper) this fosset connects as well with the inferior as with the mid frontal sulcus.

Concerning the constituents of the *inferior frontal* sulcus in recent men different opinions exist.

EBERSTALLER (l.c.) whose ideas are shared by RETZIUS stated that this sulcus is generally connected with the inferior precentral and ends after a frontal curve as an axial groove of the frontal or triangular operculum (cf. fig. 4 of my former contribution).

Frontally to this axial groove lies his *radiatus,* that may or may not be connected with it. According to ECKER [1]), SCHWALBE [2]) and KOHLBRUGGE (l.c.), however, we have to consider the whole curve including the radiatus and its branches as inferior frontal sulcus. I fully agree with KOHLBRUGGE's conception (l.c. p. 91) : "Er geht vom Präcentralis aus oder entsteht nahe an der Stelle, zieht dann zunächst in sagittaler Richtung nasalwärts, krümmt sich in Bogen oder mit scharfer Ecke ventralwärts und bildet so die Basis der Pars triangularis. Er kann den orbitalen Rand erreichen oder sich nochmals und zwar caudalwärts krümmen und den ganzen Ramus anterior fossae Sylvii mit einem Halbkreis umgeben." Such is also the most frequent relation of this sulcus in the European brains I examined.

I must, however, add to this that the large continuation of this system in nasal direction seems to be typical specially of recent men as appears from a comparative study of anthropoids, Pithecanthropus, Neanderthal and recent men.

As I pointed out before in my Chimpanzees the inferior frontal sulcus [3])

[1]) ECKER. Die Gehirnwindungen des Menschen. Braunschweig 1869.

[2]) SCHWALBE. Lehrbuch der Neurologie. 1881.

[3]) EBERSTALLER considered the fronto-orbital of anthropoids as the homologue (l.c. p. 119) of the human inferior frontal, although he remarks himself „Befremden könnte erregen dass die Furche zur Hälfte auf der Orbitalfläche gelegen ist" (l.c. p. 119), — while the inferior frontal in men never comes on the orbital surface. In my specimens of Troglodytes niger both the fronto-orbital and behind it the inferior frontal are present. Also KOHLBRUGGE contested EBERSTALLER's view. MINGAZINNI only describes two frontal fissures a superior and on infimus. The latter is, however, homologous to the human midfrontal. He overlooked the rather steep anthropoid homologue of the human frontal's inferior.

runs steeply downward after its origin from or very near the inferior precentral, reaching the orbital operculum or lateral inferior margin of the frontal lobe behind the indentation of the fronto-orbital sulcus. In Pithecanthropus it runs less steep, more obliquely, so that the inferior frontal convolution has enlarged specially on the right. At its end a small axial groove (3) of the orbital operculum appears. In Neanderthal men its curve runs still more horizontally and the inferior convolution again enlarges. The Rhodesian shows a simple relation, having only one curve, which, however, is higher and at its beginning runs more horizontally than in the ape-man. In the other Neanderthal casts the condition is less simple than in the Rhodesian [1]). The fissure shows a gradual development in the La Quina and La Chapelle (left hemispheres), and probably in the *Podbaba* cast. This development exists in a further nasal outgrowth by means of a frontal branch, perhaps indicated in the Rhodesian by a small curved fissure just before the place where the inferior frontal joins the intermediate fosset 8 (not denominated in fig. 1 Rh. r.). In the other casts, this fissure continues in the inferior frontal and forms its anterior curve or ramus anterior that may continue frontally beyond the level of the *subfrontalis* (1). This extension beyond the level of the subfrontalis fails in the Trinil cast and in anthropoids, although a double curve is indicated in the former. It may occur in recent man but the subfrontal itself may also extend correspondingly frontally here so that its anterior end may correspond again with the anterior curve of the inferior frontal fissure (see fig. 4 of my first contribution). Since in recent men the frontal curve very often runs further downward than indicated on Neanderthal casts it may again connect with the elongated axial groove of the orbital operculum (3).

In the Düsseldorf cast the r. ant. frontal. inf. is wanting on the right [2]) and on both sides the first dorsal curve shows an interruption by two bridging convolutions, the caudal one of which runs just before the inferior precentral (between this and the intermediate fissuret 8), while the anterior bridging convolution runs before fissuret 8.

This also explains why the simple fosset, that usually is 8, is changed in a curved groove, since it now lies between two parallel bridging convolutions

These bridging convolutions between the inferior and mid frontal convolution are also observed in recent men, mostly as "Tiefenwindungen", but EBERSTALLER (l.c. p. 66)

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[1]) That the simpler relation seems to occur in the Rhodesian, whose skull capacity-according to WOODWARD SMITH is 1280 cm^3, and whose encephalic index is 80,6, may be accidental, although it is interesting that RAMSTRÖM considers this the most primitive human skull hitherto found, equally primitive probably as the Heidelberg jaw. ELL. SMITH, also considers the Rhodesian as more primitive, and so does Sir ARTHUR KEITH. See RAMSTRÖM. Ueber die älteste Steinkultur in Afrika und Asien etc. Nova acta R. Soc. Scient. Upsaliens 1927, p. 27, ELL. SMITH, Rhodesian man and associated remains. Brit. Mus. Publ., 1928 p. 53. ARTHUR KEITH. The antiquity of man, 7th edition. I found a similar arrangement of the inferior frontal sulcus on the right hemisphere of an Australian aboriginal's brain, while KEITH's fig. 29 of his report on the Galilee skull (l.c. infra) shows an analogous condition on the left side of an Australian's endocranial cast.

[2]) On the left it may be indicated by a small fissure, frontally continuing (virtually) the course of 8.

observed the posterior bridging convolution in a superficial position in 24 % of his brains. The anterior bridging convolution was also observed by him (vorderste Tiefenwindung), but he does not say how often he saw it in a superficial position. Similar variations are described by CUNNINGHAM, KOHLBRUGGE, and APPLETON [1]) (l.c. see fig. 4 and p. 96). In APPLETON'S Tamil, the inferior frontal also shows that ascending branch in the direction of the mid frontal fissure, occurring on the right side of the Düsseldorf cast (between 4 and 7*c*).

The axial groove (3) on the right Düsseldorf hemisphere has become independent of 4. In recent men it is nearly always independent from the perpendicular branch of 4 (that may form the S. axialis operc. frontalis), as is easily explained by the development of the anterior horizontal branch of the fossa Sylvii, which separates the axial branch of the triangular operculum from the orbital operculum. On the other hand, groove 3 may extend frontally in recent men and connect with the descending branch of the second curve (l.c. fig. 5), a condition not hitherto observed in Neanderthal casts where the second arch of the frontalis inferior does not show that large ventral extension (see the preceding page).

Consequently, although the enlargement of the inferior frontal convolution in Neanderthal men compared with the Trinil cast and anthropoids is striking, we have no sufficient evidence that it has attained the extension of recent men, although — on the left hemispheres especially — the relations approach those of recent men.

Besides in Neanderthal casts only very vague and incomplete impressions are found on the gyrus frontalis inferior, where in recent men the rami anteriores f. Sylvii, the diagonal sulcus, and a ventral continuation of the inferior precentral are very common features. Only on the La Quina and Düsseldorf cast (left hemispheres) indications of a ventral extension of the inferior precentral are found. Of a diagonalis no trace is seen and of the rami anteriores fossae Sylvii never more than a trace (2 ?) of one groove [2]) is observed. This groove sometimes resembles a horizontal branch [2]).

I have, however, already pointed out in my first contribution that we should refrain from rapid conclusions here since SYMINGTON showed that, in recent men especially, this part of the skull never gives impressions. Still it would not be impossible if a ventral elongation of the inferior precentral and a diagonal sulcus were poorly developed in Neanderthal men, as they also fail in the so deeply impressed Trinil cast. Similarly it is not excluded that a single ramus anterior f. S. was a frequent condition, considering the fact that this is also the case in Pithecanthropus, and in 27 % of recent men, where CUNNINGHAM even found it in 41 % on the right side.

BOULE and ANTHONY considered it possible that my groove 1 might be

[1]) APPLETON. Description of two brains of natives of India. Journ. of Anat. and Physiol. Vol. 45, 1911 (fig. 4 and p. 96).

[2]) It may be that on the right hemisphere of the Düsseldorf cast the small dimple, lying above 2? is a remnant of a ram. anterior ascendens f. S. Then 2? would be a ram. horizontalis f. S.

the anterior horizontal branch, but rejected this supposition [1]). Fissure 1 is doubtlessly homologous to the fronto-orbital of apes (cf. also KEITH).

Concerning the grooves indenting the orbital margin I may be short since these relations are amply described in my former paper. They are the *fronto-marginal sulcus* (9), extending in the direction of the mid-frontal fissure (7) sometimes perhaps continuing with it as often occurs in recent men. I also pointed out that 1, the s. *subfrontalis* mihi is the homologue of the fronto-orbital of apes and may also occur in recent men, though usually smaller. It is best developed in the Rhodesian [2]) (see fig. 3 of my first contribution), where it strongly reminds us of the fronto-orbital of such Chimpanzees, where its extent on the convexity has been already reduced, which occurs more often in the Chimpanzee than in Orangs.

Resuming my results I conclude:

1⁰. that the fissuration in Neanderthal men is more humanoid than in Pithecanthropus, and that it differs from the relations in recent men;

2⁰. by the fact that Neanderthal skulls may give more impressions on the endocranial cast than recent skulls do, though less than in Pithecanthropus. In this respect both differ from anthropoid casts where hardly any impressions are visible on the frontal lobe [3]).

3⁰. by the strong development of the mid-frontal fissure, which is predominant in the relief of Neanderthal casts, and more primitive (more pithecanthropoid) in its character than in recent men, where it is mostly divided up into pieces, probably in consequence of the greater development of the foot of the mid-frontal convolution (see my first paper).

4⁰. by the fact that this groove in all casts I examined seems continuous with the precentral groove (mostly the inferior) which according to EBERSTALLER only occurs in the minority of Europeans;

5⁰. probably by a somewhat lesser development of the anterior part of the inferior frontal convolution and sulcus.

I will not end without adding a few words concerning SYMINGTON's criticism (l.c.) on the work of BOULE, ANTHONY and ELL. SMITH, the reliability of which he doubted on account of the failing impressions on endocranial casts of recent men. I do not doubt the accuracy of SYMINGTON's results in recent men, the less so as also the cast of the Aurignac Homo sapiens fossilis (in contrast to the Předmost cast; TILNEY and RILEY [4])) shows no impressions whatever on the frontal lobe. This, however, gives us

[1]) BOULE and ANTHONY Neopallial morphology as studied from endocranial casts of fossil men. Journ. of Anat. and Phys. Vol. 51, 1917.

[2]) In an australian aboriginal's brain, which I had the opportinity to examine, this sulcus was just as strongly developed on both sides as in the Rhodesian.

[3]) In this respect also the ape man of Java approaches more the Neanderthal men than the anthropoids do.

[4]) See their photographs and description of this cast in "The brain from Ape to Man". Vol. 2 fig. 408—414 and p. 918—922.

PLATE I.

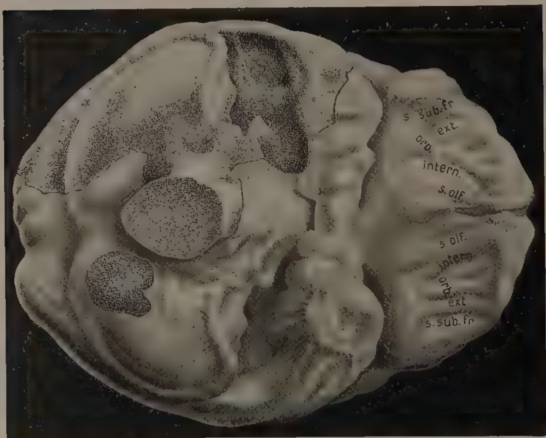

Fig. 1
Basal aspect

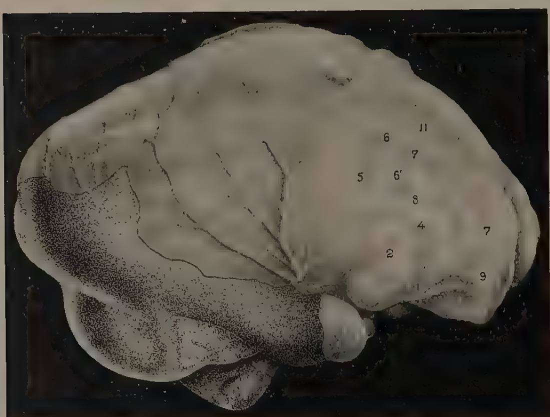

Fig. 2
Right
hemisphere

Endocranial cast of the Rhodesian skull. — 9 in fig. 2 refers to the sulcus behind it.

PLATE [illegible]

Fig. 1
Düsseldorf
cast

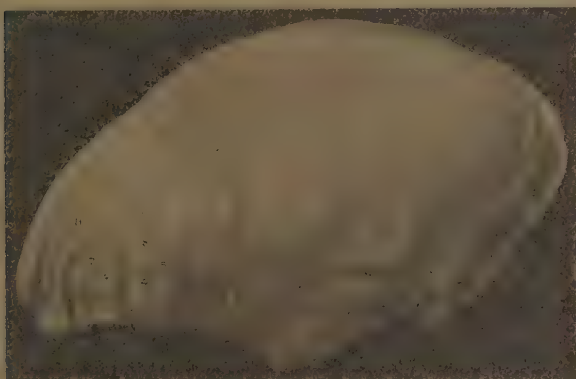

Fig. 2
La Quina
cast

Fig. 3
La Chapelle
cast

For the identification of fissure 1 in the La Chapelle cast see text [illegible]

Fig. 1
Basal aspect

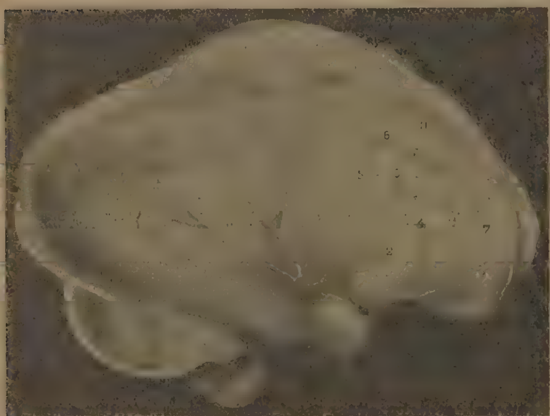

Fig. 2
Right
hemisphere

Endocranial cast of the Rhodesian skull. — 9 in fig. 2 refers to the sulcus behind it.

C. U. ARIËNS KAPPERS: FURTHER COMMUNICATIONS ON THE FISSURES OF THE FRONTAL LOBES IN NEANDERTHAL MEN.

PLATE II.

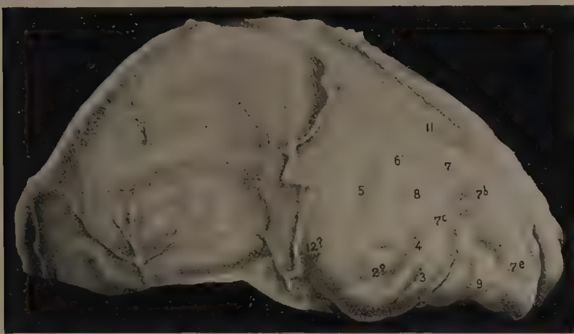

Fig. 1
Düsseldorf
cast

Fig. 2
La Quina
cast

Fig. 3
La Chapelle
cast

For the identification of fissure 1 in the La Chapelle cast see textfigure 2.

no right to doubt the value of impressions occurring on Neanderthal casts, the less so since they are still more pronounced in the ape man from Java, and can be easily correlated with the latter.

The question is: why are the endocranial impressions more pronounced in the Neanderthal and ape-man than in Homo recens?

According to SCHWALBE they are present in all animals except the Orang Utan, Chimpanzee and Gorilla (l.c. p. 382) and in newborn children are indicated, though slightly, on nearly all the skull bones (l.c. p. 380).

Apparently a special pressure on the skull is responsible for this. This may also explain their constant appearance on the orbital surface in the erect going men, even in recent men.

That the impressions on the frontal lobes in Neanderthal casts may be trustworthy guides for studying the fissuration is, moreover, confirmed by the fact that their study, combined with that of the Pithecanthropus impressions yields such conclusive results, as shown in this and my former paper. I may add that this also appears from the studies of Sir ARTHUR KEITH [1]) on the Piltdown and Australian endocranial casts (l.c. Vol. II, fig. 221 and 222) where this author also found indications of the frontal fissures and from his studies on the Galilee and Gibraltar casts [2]). His conclusions in several aspects agree with my results, obtained before I had any knowledge of those of KEITH, and by the study of other casts than those used by the English anthropologist.

It is a great satisfaction to me not only to confirm several of BOULE and ANTHONY's and KEITH's results, but to extend their data and correlate them with the relations in the ape man of DUBOIS.

EXPLANATION OF FIGURES:

1 = fronto-orbitalis or subfrontalis mihi.
2 = ramus anterior fossae sylvii.
3 = axial groove of the orbital operculum.
4 = frontalis inferior.
5 = precentralis inferior.
6 = connection between the latter and the frontalis medius.
6' = ventral branch of the frontalis medius.
7 = frontalius medius.
8 = intermediate fosset of the midfrontal convolution.
9 = fronto-marginal.
10 = sulcus bordering the rostrum.
11 = frontalis superior.
12 = subcentralis anterior;
c = ventral end of the centralis.

1) KEITH. The antiquity of man. 7th Edition, 1928.

2) KEITH. A report on the Galilee skull. Publications of the British school of Archaeology in Jerusalem, 1927.

Physics. — *On the resistance-hysteresis phenomena of tin, lead, indium and thallium at the temperature of liquid helium.* By W. J. DE HAAS and J. VOOGD. (Comm. N⁰. 191*d* from the Physical Laboratory at Leiden).

(Communicated at the meeting of June 30, 1928).

§ 1. *Object of the research.*

It has been found that with the magnetic disturbance of the super-conductivity of tin and mercury hysteresis phenomena appear in the change of resistance.

If the magnetic field is strengthened the resistance comes back at a higher strength of field than that at which it disappeared in a diminishing magnetic field [1]). We thought it would be of interest to investigate whether all super-conducting metals showed this phenomenon. We therefore examined the magnetic transformation figure in an indium, a thallium and a lead wire. We also examined a tin wire so made that we might expect an analogy of the magnetic transition figure with that of the mercury wires previously examined.

§ 2. *Tin. Magnetic transition figure of Sn.-17-1927.*

This wire was made from "Kahlbaum" tin, which was forced in liquid state into a thick walled glass capillary tube and slowly cooled from one end.

In this way we hoped that the wire would be composed of a few crystals, if possible of one single crystal. The latter was not attained. This was probably due to the too great heat capacity of the thick glass wall; when using a thin-walled capillary tube we succeeded in obtaining single crystal wires (see § 3).

The glass was etched off with hydrofluoric acid, which did not effect the tin wire. The wire thus obtained was mounted without tension in an ivory holder.

The four leads for the resistance measurements were fused on with WOOD's metal.

The resistance measurements were made in the usual way with a thermoforce-free DIESSELHORST compensation apparatus, with a ZERNIKE

[1]) G. J. SIZOO, W. J. DE HAAS and H. KAMERLINGH ONNES. Leiden Comm. 180*c*. W. J. DE HAAS, G. J. SIZOO and H. KAMERLINGH ONNES. Leiden Comm. 180*d*.

galvanometer as a null instrument, while the magnetic fields (always parallel to the axis of the wire) were obtained with a coil C consisting of three layers of copper ribbon wound over a length of 38 cm. This coil was placed round the outer cryostat glass. In the tables we give the homogenity of the magnetic field along the wire for each of the wires.

In fig. 1 the magnetic transition figure of *Sn*-17-1927 is given, the data of which are found in Table 1. As may be seen, the character of this curve corresponds completely with the mercury figures formerly determined. Regarding the striking analogy with the transition figures of mercury wires we wish to add the following remarks: *Sn*-17-1927 is longer than the wires which have latterly been used in the investigation of the hystersis phenomena in mercury. This investigation had taught us that the unavoidable slight inhomogenity of the magnetic field in the direction of the axis has an influence upon the transition figure, in the sense that the upper part of the figure in relation to the under part is shifted towards the high fields. This shifting can bee seen in the transition figure of *Sn*-17-1927 in complete analogy to the mercury wires.

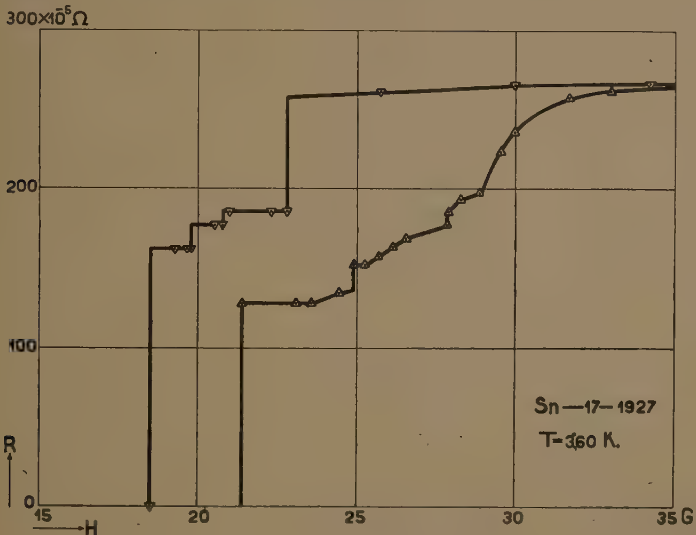

Fig. 1.

§ 3. *Indium. Magnetic transition figure of In-1-1928.*

The indium used was supplied by ADAM HILGER Ltd.

The liquid indium was forced into a thin-walled glass capillary tube and cooled from one end. As we had found with tin, that it was possible to make a mono-crystalline wire in this way, we hoped to succeed in doing so with indium.

TABLE 1.

Sn-17-1927
Date: 17-6-1927

$P_{helium} = 393$ mm.

length = 19 mm. diam. 0.038 mm.
$R_{4.2} = 0.00298\ \Omega$
Current 2.5 mA.
$T = 3°.60$ K.
Homogenity 0.998

H	R	
0	0	
21.40	0	
23.07	0.00128	jump ↑
23.11	0.00128	
23.54	0.00128	
24.44	0.00134	
24.91	0.00152	jump ↑
25.25	0.00152	
25.68	0.00157	
26.11	0.00163	
26.54	0.00168	
27.82	0.00177	
27.86	0.00185	
28.29	0.00193	
28.89	0.00197	
29.53	0.00224	
29.96	0.00236	
31.67	0.00257	
32.96	0.00262	
64.20	0.00269	
50.12	0.00268	
38.56	0.00268	
34.20	0.00265	
29.96	0.00265	
25.72	0.00260	
22.77	0.00185	jump ↓
22.30	0.00185	
20.97	0.00185	
20.75	0.00177	jump ↓
20.54	0.00177	
19.77	0.00162	jump ↓
19.69	0.00162	
19.26	0.00162	
18.49	0	jump ↓

After the glass had been etched away the wire was mounted without tension. For the resistance measurements, four indium leads were fused on.

In fig. 2 and table 2 we give the magnetic transition figure which shows a wide hystenesis. From the fact that the resistance disappears with one jump we should deduce that *In*-1-1928 has a strongly marked single-

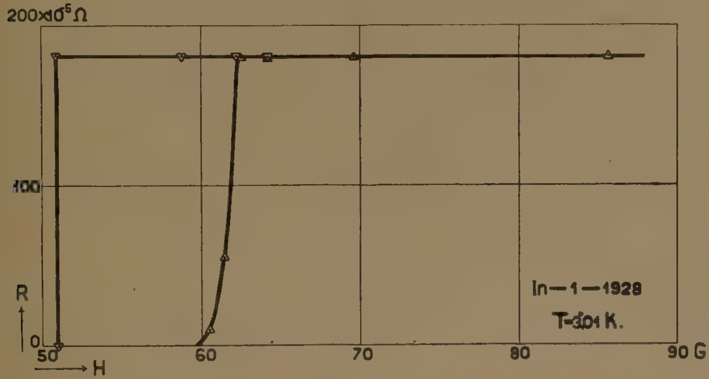

Fig. 2.

TABLE 2.

In-1-1928
Date: 27-6-1928

P_{helium} = 166.4 mm.

length = 14 mm. diam. = 0.075 mm.
$R_{3.42}$ = 0.001806 Ω
Current: 10 mA.
T 3°.01 K.
Homogenity 0.999

H	R	
0	0	
60.56	0.000106	
61.42	0.000560	
62.49	0.001806	
64.20	0.001806	
69.66	0.001806	
85.60	0.001806	
107.0	0.001806	
64.20	0.001806	
62.17	0.001806	
58.85	0.001806	
50.93	0	jump ↓

crystalline character. We have not tested this in any other way, however, and as in the mounting the wire became slightly bent we do not yet wish this transition figure te be regarded as typical of perfect single crystalline wires.

§ 4. *Lead. Magnetic transition figure of Pb-1-1928.*

The wire of Kahlbaum lead was made and mounted exactly like *In*-1-1928. As is well known, very small deformations will cause lead even at ordinary temperature, to re-crystallize to a conglomerate of small crystals. It is therefore probable that we had to deal with a wire built up of a number

TABLE 3.

Pb-1-1928 length = 10 mm diam. = 0.045 mm.
Date: 27-6-1928 Current = 20 mA.
P_{helium}: 775.9 mm. $T = 4^{\circ}.22$ K.
Homogenity 0.999.

H	R	
0	0	
563.9	0	
600.3	0.000076	
605.6	0.000277	
608.8	0.000355	
612.0	0.000388	
618.5	0.000401	
629.2	0.000434	
759.7	0.000478	
843.2	0.000489	
622.7	0.000430	
618.5	0.000423	
602.4	0.000382	
588.5	0.000326	
579.9	0.000192	
579.4	0. 0	heavy fall

of small crystals, which is also indicated by the transition figure (Fig. 3). The ascending curve is rounded off at the top, and even at the highest magnetic field that we could attain with coil *C* (850 Gauss) it continued to rise.

The descending curve is also rounded off.
It is possible that this is due to the fact that with coil C we could not

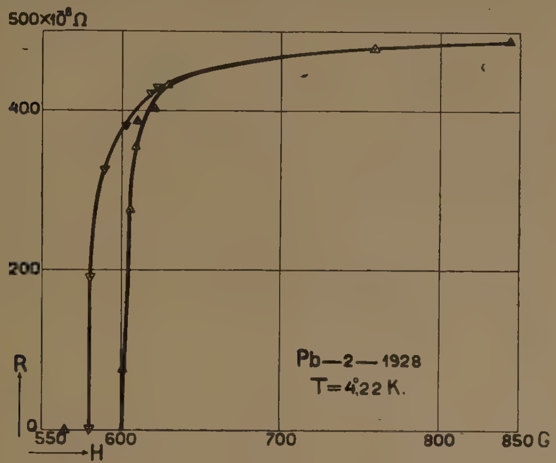

Fig. 3.

raise the magnetic field above 850 Gauss. However this may be, we think it possible that the micro-crystalline character of *Pb*-1-1928 was also one of the factors that contributed towards the rounding of the curves (see § 5).

§ 5. *Thallium. Magnetic transformation figure of Thallium-1-1928.*

As thallium oxidises very rapidly we made a resistance from the not very pure thallium from KAHLBAUM resembling the mercury resistances formerly

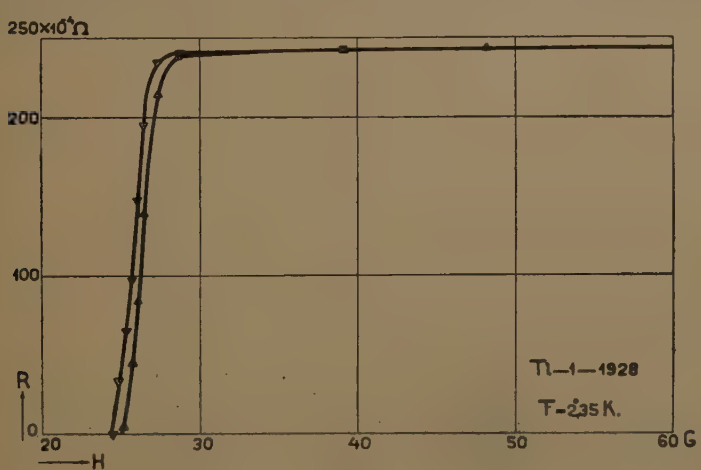

Fig. 4.

TABLE 4.

Tl-1-1928 length $=23$ mm. diam. $=0.10$ mm.
Date: 27-6-1928 $R_{4.2}=0.2451\ \Omega$
Current 10 mA.
$P_{helium}=44.4$ mm, $T=2^{\circ}.35$ K.
Homogenity 0.998

H	*R*	
0	0	
24.48	0	
25.08	0.00042	
25.68	0.00446	
26.07	0.00841	
26.45	0.01387	
27.39	0.02146	
28.68	0.02383	
39.08	0.02429	
47.12	0.02432	
64.20	0.02436	
39.08	0.02429	
28.68	0.02403	
27.39	0.02343	
26.45	0.01946	
26.07	0.01484	
25.68	0.00971	
25.25	0.00650	
24.82	0.00339	
24.40	0.00002	
23.97	0	

used (type II of Leiden Comm. 180*d*). The liquid thallium was forced into a glass capillar tube, in the widened parts of which at each end two platinum wires were fused as contacts for the resistance measurements.

The liquid thallium was then rapidly cooled off, so as to procure a multi-crystal wire.,The transformation figure (Fig. 4) again shows two rounded off lines, which also at the highest fields at which measurements were made

still rose, though only slightly. As in lead, we think that the micro-crystalline composition of Tl-1-1928 was one of the causes, if not the sole cause, of this.

§ 6. *Conclusion.*

The experiments described above show that hysteresis phenomena appear in all superconducting metals. As in the earlier experiments, we get the impression that the phenomena occur in the purest form only when there are large crystals and when there is a homogeneous magnetic field.

In conclusion we wish to express our thanks to Mr. W. H. Capel and Mr. P. M. van Alphen for their assistance in the observations.

Physics. — *On the super-conductivity of Gallium.* By W. J. DE HAAS and J. VOOGD. (Communication N⁰. 193*b* from the Physical Laboratory at Leiden).

(Communicated at the meeting of September 29, 1928).

§ 1. The behavior of the electric resistance of gallium placed in liquid helium was examined by TUYN and KAMERLINGH ONNES. They generated an induction current in a gallium ring, and showed that the gallium they examined was not super-conducting at $T = 1^{\circ}.6$ K. in a field of 17 Gauss.

As the place that gallium takes in the periodic system (three electrons in the outermost shell, 18 in the following one) is of importance to the problem of super-conductivity we thought it would be of interest to investigate the course of the resistance by direct resistance measurements down to the lowest temperature attainable and only in the earth magnetic field, which is not compensated.

§ 2. The gallium we used was supplied by ADAM HILGER LTD. According to the firm's spectroscopic analysis, this gallium contained in impurities 0.16 % indium, 0.10 % zink, 0.01 % lead and a trace of sodium.

By forcing the liquid gallium into a glass capillary tube and letting it crystalize and then etching away the glass we obtained a wire, of which we made the resistance *Ga-2-'28*. As tension wires for the resistance measurements gallium wires were fused on just within the two extremities. The resistance thus contrived was mounted tension free upon an ivory rod.

We also made a resistance *Ga-3-'28* in the same way, of gallium with an impurity of 0.3 % of indium.

The resistance measurements were made in the usual way with a DIESSELHORST compensation apparatus. As the value of the resistance, by which should be understood the quotient of potential difference and strength of current, proved in both resistances at a low temperature to be strongly influenced by the strength of the current causing the potential difference, we made measurements with various strengths of current.

The results will be found in the accompanying tables and figures.

§ 3. The resistances, even at the lowest temperature that we could attain, did not become super-conducting.

At the same time the curve (Fig. 1) has all the characteristics of super-conductivity and we may presume that at a temperature slightly lower than $1^{\circ}.1$ K. gallium would become super-conducting. Gallium would then be

TABLE 1.

Ga—2—'28				
T	p_{helium} in mm.	$37^1/_2$ mA.	10 mA.	$2^1/_2$ mA.
293		1.5698 Ω		
273.09		1.4680		
4.20	758	0.003749	0.003750	
3.42	311	0.003728	0.003723	
3.37	289	0.003727	0.003728	
3.00	165	0.003719	0.003710	0.00373
2.50	62.3	0.003711	0.003709	0.00370
1.98	16.9	0.003701	0.003695	0.00367
1.35	2.25	0.003673	0.003643	0.00355
1.1	0.46	0.003590	0.003364	0.00268

TABLE 2.

Ga—3—'28 [1]		
T	p_{helium} in mm.	$37^1/_2$ mA.
293		0.1066 Ω
4.20	758	0.000068
3.42	311	0.000068
3.37	289	0.000066
3.00	165	0.000060
2.50	62.3	0.000048
1.98	16.9	0.000034
1.35	2.25	0.000014
1.1	0.46	0.000006

the 6th super-conductor [2]). It is another question, however, whether this

[1]) This wire was more sensitive to current than *Ga*—2—'28. We hope to return to the question of current sensitivity at a future time.

[2]) Recently however Dr. W. MEISSNER found Tantal also to be super-conduction Phys. Zeitschr. **29**. 1928. p. 897.

super-conductivity is connected with the trace of indium in the gallium. Fresh experiments with more pure gallium will be needed to put this to the test.

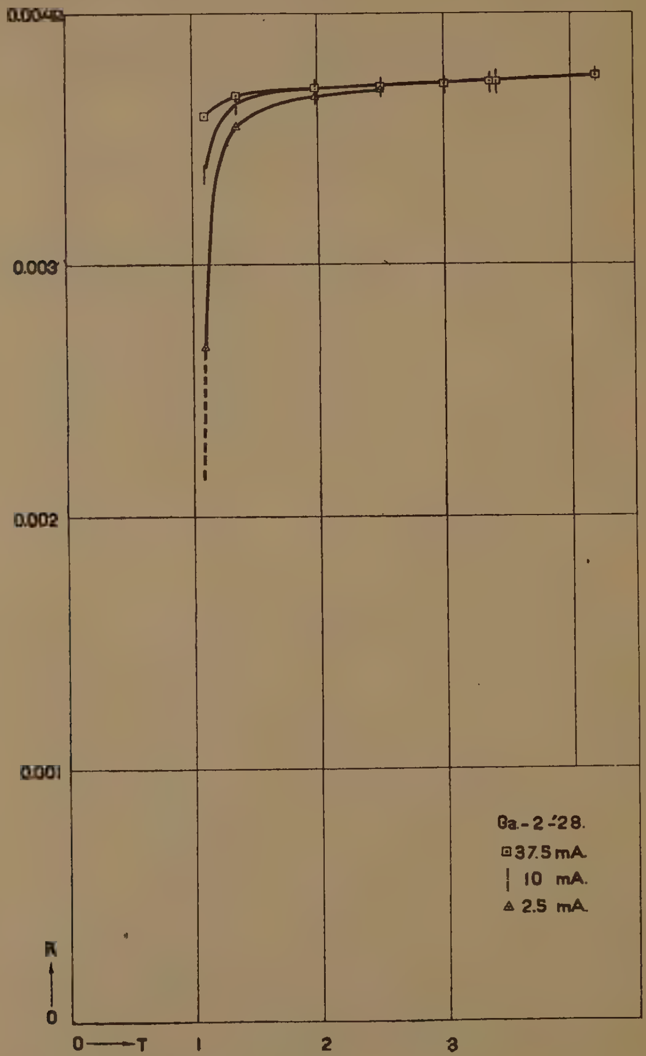

Fig. 1.

§ 4. *Abnormal phenomena.*

We wish to point out the following abnormal phenomena.

1. The fall of the resistance is of an abnormal character. As can be

seen in the figures, the resistance-temperature lines are very curved at low temperatures, the curve extending over a range of temperature of several

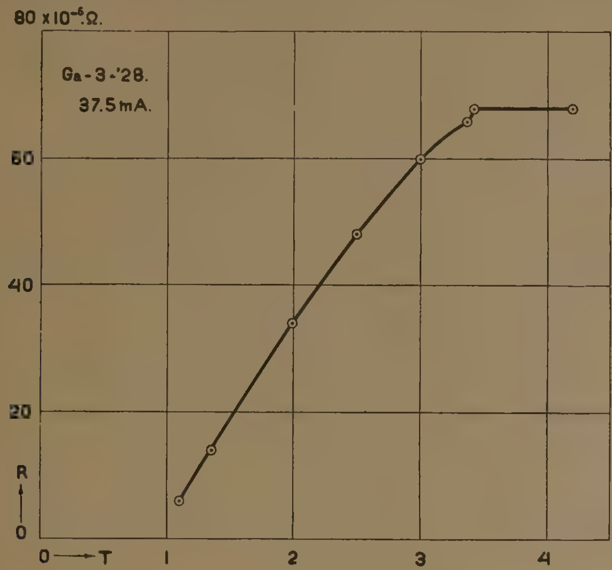

Fig. 2.

degrees. (With ordinary super-conductors the curve only extends over a few 100ths of a degree).

2. The resistances, as stated above, are very sensitive to current.

3. At the transition point of indium (3°.42 K.) in *Ga-3-'28* a slight extra decrease of resistance appears. Fig. 2.

4. If the behavior of the two resistances is compared it is found that *Ga-3-'28*, with more indium impurity, decreases much more in the temperature range 4.2 K. to 1.1 K. than *Ga-2-'28* does.

In the same way (contrary to MATTHIESEN's rule) the resistance at 4.2 K. as compared to that at ordinary is much more reduced in *Ga-3-'28* (the impure) than in *Ga-2-'28*.

5. The result from *Ga-3-'28* seem to show a similarity with those found by TUYN and KAMERLINGH ONNES for some cadmium resistances.

Physics. — *New Super-Conductors.* By EDM. VAN AUBEL, W. J. DE HAAS and J. VOOGD. (Comm. N⁰. 193*c* from the Physical Laboratory at Leiden.)

(Communicated at the meeting of September 29, 1928).

§1. *Object of the experiments.*

The behaviour of the electric resistance of compounds of two metals at low temperatues, has not so far been much investigated.

It is known that these compounds at ordinary temperature are sharply distinguished from alloys of the same metals, not in chemical combination, by the extreme values for specific conductivity.

Further, the Röntgen analysis of compounds of two metals has shown that the atoms of the components are arranged regularly in the lattice.

On the basis of these data it seemed to us that it would be of interest to examine the behaviour of the electric resistance at low temperatures.

We wished to pay special attention to compounds in which one of the components belonged to the super-conducting metals.

We examined rods of Cu_3Sb, Ag_3Sb, Ag_3Sn, Cu_3Sn, Bi_5Tl_3, $SbSn$ and further a rod of probably the compound of Sb and Sn containing 40.21 % Sb in the proportion two Sb to three Sn [1]).

All the rods were prepared by one of us (VAN A.) in the physical laboratory at Ghent.

§ 2. *Resistance measurements between 0° C. and —259° C.*

Although the real object of our investigation was the behaviour of compounds in liquid helium, we determined the resistances over the whole of the above range of temperature. We give the results below, practically without comment.

At each extremity of the rods two wires were soldered on for the resistance measurements. The rods were then placed in open guard-tubes and mounted in the cryostat.

The temperatures were attained by baths of methyl chloride, ethylene, oxygen and hydrogen.

The resistance measurements were made in the usual way with a DIESSELHORST thermoforce-free compensation apparatus.

In Table I we give the values for the resistances at 0° C. before and after the measurements. Only in Cu_3Sn does the value seem to have

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[1]) According to KONSTANTINOW and SMIRNOW the compounds $SbSn$ and Sb_2Sn_3 exist. According to BRONIEWSKI and SLIKOWSKI only the compound Sb_2Sn_3 exists. BRONIEWSKI et SLIKOWSKI, C. R. Paris. 11—6—1928 p. 1615. Revue de métallurgie Paris 6—1928, p. 312.

changed with time. The resistance of Ag_3Sb and of Ag_3Sn changed through a potential wire breaking during the measurements. This had to be repaired, an operation, which slightly shifted the contacts.

TABLE 1.

	Resistance at 0° C. before measurement in liquid Helium	Resistance at 0° after measurement in liquid Helium	Specific resistance $\times 10^3$
Cu_3Sb	0.005150 Ω	0.005146 Ω	0.770
Ag_3Sb	0.020796		
	0.020987	0.020978	1.417
Ag_3Sn	0.003702		
	0.003698	0.003698	0.190
Cu_3Sn	0.000802	0.000826	0.100
Bi_5Tl_3	0.011049	0.011042	0.583
$SbSn$	0.004361	0.004360	0.302
Sb_2Sn_3	0.002575	0.002574	0.250

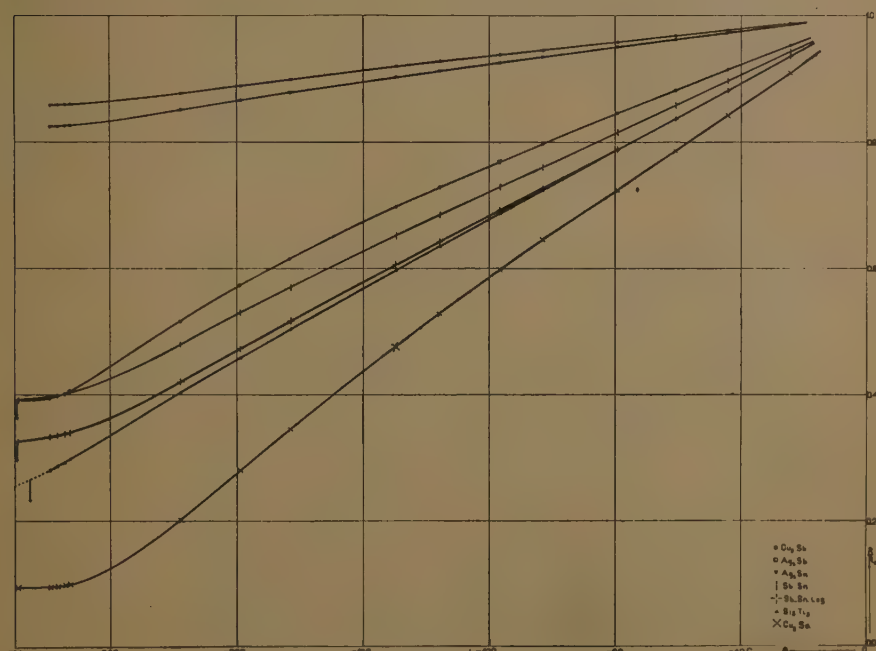

Fig. 1.

From these data and the dimensions of the rods we determined the specific resistances. The values found are given also in Table 1 (accuracy specific resist 1 %).

In Table 2 the values for the resistance, divided by the resistance at 0° C. are given for the different temperatures. In Fig. 1 the course of the resistances is given graphically.

TABLE 2.

Θ	Cu_3Sb	Ag_3Sb	Ag_3Sn	Cu_3Sn	Bi_5Tl_3	$SbSn$	Sb_2Sn_3
—24.48					0.9344		
—24.50				0.9081			
—24.51		0.9851					
—24.52	0.9876					0.9424	
—24.53							0.9333
—24.54			0.9528				
—44.27	0.9771	0.9728	0.9148	0.8417			
—44.28					0.8811	0.8962	0.8808
—60.94	0.9682	0.9621	0.8819				
—60.95				0.7852	0.8364		0.8363
—60.96						0.8571	
—79.29	0.9583				0.7871	0.8149	
—79.31							0.7885
—79.32		0.9501		0.7224			
—79.34			0.8453				
—103.09	0.9460		0.7967	0.6453	0.7229	0.7599	0.7265
—103.10		0.9346					
—116.61	0.9386		0.7686	0.5976	0.6864	0.7286	
—116.62		0.9254					0.6917
—135.72					0.6342		
—135.73				0.5271			
—135.74	0.9277	0.9122	0.7274			0.6840	0.6417
—149.65	0.9195		0.6963				
—149.66		0.9023		0.4750		0.6512	

TABLE 2 (*Continued*).

Θ	Cu_3Sb	Ag_3Sb	Ag_3Sn	Cu_3Sn	Bi_5Tl_3	$SbSn$	Sb_2Sn_3
—149.67					0.5962		0.6052
—183.10	0.8985					0.5699	0.5163
—183.11				0.3456	0.5032		
—183.24		0.8779	0.6150				
—198.93	0.8886	0.8657	0.5729		0.4582		0.4728
—198.94				0.2801		0.5297	
—217.86							0.4199
—217.91	0.8767	0.8504	0.5160				
—217.92						0.4802	
—217.93				0.2019			
—217.94					0.4030		
—252.65				0.0998			
—252.66	0.8596	0.8262	0.4054		0.2978	0.4029	0.3391
—254.38	0.8591	0.8255	0.4015	0.0979	0.2927	0.4007	0.3368
—256.62	0.8587	0.8247	0.3972	0.0961	0.2865	0.3984	0.3344
—258.99	0.8582	0.8242	0.3937	0.0947		0.3964	0.3323
—259.00					0.2804		

§ 3. *Graph of Table 2.*

Our results show that the resistances of Cu_3Sb and Ag_3Sb depend little upon the temperature. This was to be expected by analogy with MATTHIESEN's rule on account of the relatively high values of the specific resistance of these substances.

Further we found that the resistance of all the rods decreased less than that of their components.

Finally we wish to draw attention to the fact that all the resistance lines show an inflection point. This phenomenon is known to occur in some simple conductors.

§ 4. *Resistance measurements in liquid helium.*

For this, the most important part of the investigation, the resistances of Ag_3Sn, Cu_3Sn, Bi_5Tl_3, $SbSn$ and Sb_2Sn_3 were mounted in the Helium cryostat.

In Tables 3, 4, 5, 6 and 7 the values for R/R_0 at the different temperatures

TABLE 3.

Bi_5Tl_3		
T	p_{helium}	R/R_0
4.2	760	0.0000

TABLE 4.

Sb_2Sn_3		
T	p_{helium}	R/R_0 170 m.A.
4.20	760	0.3257
4.10	689	0.3196
4.00	622	0.0637
3.95	589	0.0062
3.80	500	0.0000

TABLE 5.

$Sb—Sn$				
T	p_{helium}	R/R_0		
		170 m.A.	85 m.A.	17 m.A.
4.20	760	0.3917		
4.10	689	0.3911		
4.00	622	0.3672		
3.95	589	0.3080	0.2995	0.277
3.80	500	0.0927		
3.75	473	0.0616		
3.73	463	0.0537		
3.65	421	0.0349		
3.55	369	0.0261		

TABLE 6.

Ag_3Sn				
T	p_{helium}	R/R_0		
		147 m.A.	37 m.A.	10 m.A.
4.22	775	0.3865	0.3864	
3.42	311	0.3857		
3.00	165	0.3492		0.336
2.52	64	0.2838		0.250
1.99	17.5	0.2230		0.178
1.36	2.3	0.1668		0.111

TABLE 7.

Cu_3Sn					
Date	T	p_{helium}	R/R_0		
			140 m.A.	80 m.A.	14 m.A.
29—11—'27	4.20	760	0.095^0		
„	3.80	500	0.094^9		
„	3.55	369	0.094^9		
„	2.89	136	0.095^2		0.09^9
20—1—'28	1.80	10.0		0.093	0.09^1
„	1.54	5.6		0.093	0.09^4
„	1.31	1.9		0.092	0.09^0

are given. In the regions of rapid decrease of resistance OHM's law becomes invalid on account of the sensitivity of the resistance to current. Where this was the case we have usually made measurements with various strengths of current. As value for the resistance, in these cases the quotient of potential difference and strength of current is taken as usual. The results are seen in Fig. 2.

§ 5. We found, accordingly, that in the resistances of Bi_5Tl_3, $SbSn$ and Sb_2Sn_3 the phenomenon of supra-conductivity is developed. *It is very*

surprising that the rod of Bi_5-Tl_3 becomes super-conducting even above the boiling point of helium.

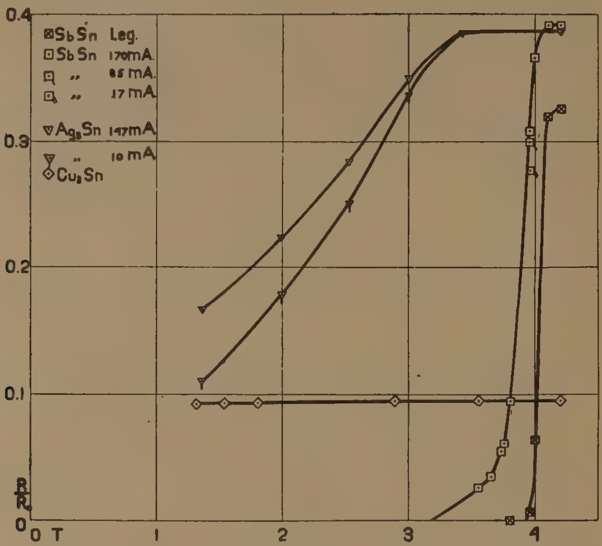

Fig. 2.[1])

Thallium only becomes super-conducting at 2.47° K. and bismuth retains its full resistance certainly down to 1.5° K.

We thus find an enormous elevation in the transition point. We might suspect this to be due to an impurity of lead, of which the metals used for the compounds certainly contained a slight amount. As, however, this amount is not sufficient in either metal to make the resistance disappear above 4°.2 K. [2]) we do not think it probable that the high transition point of Bi_5Tl_3 is due to this.

The bevaviour of $SbSn$ and of Sb_2Sn_3 are in support of this.

Here also the fall of resistance begins at higher temperatures than with pure tin (3°.8 K.). The tin used ("Kahlbaum") is very pure.

As the antimony-rich Sb-Sn becomes superconducting at a lower temperature than the antimony-poorer Sb_2Sn_3, it is impossible to assume an influence of possible lead impurity in the antimony upon the shifting of the transition point.

Thus we see that in these three substances super-conductivity is more easily induced than in their super-conducting components.

This peculiarity in two super-conducting components had already been

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[1]) ⊠ *Sb Sn leg.* in Fig. 2 should be after BRONIEWSKI and SLIKOWSKI Sb_2Sn_3.

[2]) *Bi* en *Tl* from KAHLBAUM from which the preparation was made have been examined on a former occasion.

observed by KAMERLINGH ONNES with an amalgamated tin-foil. That it is not a general rule, however, is proved by the course followed by the resistance in Ag_3Sn and Cu_3Sn.

With Ag_3Sn there was a relatively large fall of the resistance and a great sensitivity to current. It is possible that these phenomena were brought about by an excess of tin in the rod. In that case pure Ag_3Sn would be able to retain its residual resistance down to the lowest temperatures.

This question must be decided by further experiments, but at any rate it is certain that at the temperatures investigated, this tin-rich compound did not become super-conducting.

With Cu_3Sn this fact was even more clearly demonstrated. Here the residual resistance remains constant down to the lowest temperature, and the metal behaves like a normal non-super-conductor.

In conclusion we have great pleasure in thanking Mr. W. H. CAPEL for his assistance in the observations and the calculations.

Physics. — *Ein aus zwei Nicht-Supraleitern zusammengesetzter Supraleiter.* Von W. J. DE HAAS, EDM. V. AUBEL und J. VOOGD. (Mitteilung N⁰. 197*a* aus dem Physikalischen Institut, Leiden).

(Communicated at the meeting of January 26, 1929)

§ 1. Die Resultate der Untersuchung des elektrischen Widerstandes einiger Metallverbindungen im flüssigen Helium machte es in unseren Augen wünschenswert auch eine Kombination von Gold und Wismuth zu untersuchen [1]).

Bei der Untersuchung von Metallverbindungen hatten wir nämlich den Eindruck bekommen, dass eine Kombination eines Supraleiters mit einem Nicht-supraleiter leichter supraleitend wird als die supraleitende Komponente, wenn die Atomgewichte der beiden Metalle wenig von einander abweichen.

Will man eine supraleitende Kombination von Nicht-supraleitern herstellen, so kommen, mit Rücksicht auf die obenstehende Bemerkung, dafür Gold und Wismuth in Betracht. Nun kommt im System Gold-Wismuth keine Verbindung vor. Nach dem Schmelzpunktsdiagramm ist Wismuth bis $4\frac{1}{2}\,\%$ in Gold löslich und Gold gar nicht in Wismuth [2]). In dem aus $82\,\%$ *Bi* und $18\,\%$ *Au* bestehenden Eutektikum findet man also neben einander kristallisiert eine feste Lösung von $4\frac{1}{2}\,\%$ *Bi* in *Au* und metallographisch reines Wismuth. Bei niedrigerer Temperatur bekommt im festen Zustand die feste Lösung einen etwas höheren Gehalt an Wismuth auf Kosten des reinen Wismuths.

Für unsere Untersuchungen benutzten wir an erster Stelle die eutektische Mischung.

§ 2. Einer von uns (V. A.) stellte in Gent ein Stäbchen der eutektischen Mischung her. Hierzu verwendete er reines Gold von Heraeus und reines Wismuth von Matthey. Das Stäbchen hatte einen Gehalt von $17{,}9\,\%$ an Gold.

Da das Eutektikum bei $240°$ C. schmilzt war es sehr leicht für die Widerstandsmessung vier Kupferdrähte anzulaschen, wobei also der Gebrauch von Löte gänzlich vermieden wurde.

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[1]) Verslagen Kon. Akad. v. Wetensch. Amsterdam **27** N⁰. 7; Comm. Leiden N⁰. 193*e*.

[2]) Sieh. Int. Crit. Tables, Vol. **2**. S. 424.

Von dem so hergestellten Widerstand *Au—Bi* I wurde der Widerstand in der üblichen Weise bei den verschiedenen Heliumtemperaturen gemessen. Die Resultate findet man in Tabelle I. Die vorausgesetzte Möglichkeit, dass eine Gold-Wismuth-Kombination supraleitend werden könnte stellte sich als Wirklichkeit heraus.

TABELLE I. *Au — Bi — I.*

T	p in mm Hg	R/R_0	i in mA	Datum
4.195	756	0.7469	28	11 December 1928
3.337	278	0.7472	28	„
1.997	17.7	0.0000	28	„
2.650	85.8	0.7442	10	„
2.508	63.5	0.7451	„	„
2.400	49.6	0.7451	„	„
2.266	36.1	0.7451	„	„
2.130	25.4	0.4808	„	„
2.118	24.5	0.1126	„	„
2.099	23.4	0.0409	„	„
2.088	22.6	0.0181	„	„
2.041	19.9	0.0000	„	„
4.206	765	0.7453	12	20 December 1928
2.996	164	0.7450	„	„
2.487	61.1	0.7441	„	„
2.177	28.7	0.7410	„	„
2.135	25.7	0.6657	„	„
2.116	24.3	0.2673	„	„
		0.496	93	„
2.093	22.9	0.0350	12	„
		0.0802	93	„
2.056	20.8	0.0086	12	„
		0.0130	93	„

Die Versuche wurden wiederholt mit einer eutektischen Mischung von Gold, uns freundlichst von Dr. MICHELS, Amsterdam, zur Verfügung

gestellt, und Wismuth von der Firma HILGER, Londen. Nach Angabe der Firma ist das Wismuth sehr rein. Chemisch konnte eine Spur Silber angezeigt werden, spektrographisch eine Spur Kupfer.

Die zuletzt verschwindenden Linien 3273,967 und 3247,550 (Hasbach) waren im Bogenspektrum nur als sehr schwache Linien sichtbar.

Blei war nur spurweise vorhanden, wie aus dem Auftreten der schwachen Linien 4057,84; 3683,47; 3639,53; 2833,06 und 2614,20 hervorging.

Im Kryostat wurden aufgestellt die Komponenten der eutektischen Mischung (*Au* I 1928 und *Bi* II 1928) und ein Stäbchen der eutektischen Mischung.

Das Wismuth war in die Form eines gegossenen Stäbchens gebracht und genau auf die nämliche Weise behandelt als das gegossene Stäbchen der eutektischen Mischung (*Au—Bi* II 1928).

Die Resultate der Widerstandskurven sind in den Tabellen II, III und IV sowie in Fig. I dargestellt.

TABELLE II. *Au — Bi — II.*

T	p in mm Hg	R/R_0	i in mA	Datum
4.221	775	0.1665	12	20 December 1928
3.606	397	0.1665	„	„
3.390	299	0.1664	„	„
3.208	230	0.1663	„	„
3.500	62	0.1659	„	„
2.248	34.5	0.1660	„	„
2.192	29.7	0.1659	„	„
2.151	26.9	0.1659	„	„
2.119	24.6	0.1657	„	„
2.068	21.5	0.1640	„	„
2.040	19.9	0.1623	„	„
2.004	18.2	0.1575	„	„
1.973	16.5	0.1541	„	„
1.915	14.1	0.0000	„	„
„	„	0.0000^5	92	„
1.946	15.3	0.1279	12	„
1.936	15.0	0.0590	12	„
„	„	0.1047	92	„

TABELLE III. *Au — I —* '29.

T	p in mm Hg	R/R_0	i in mA	Datum
4.221	775	0.006765	12	20 December 1928
2.750	105	0.006818	„	„
2.500	62	0.006828	„	„
1.23	1.1	0.006924	„	„

TABELLE IV. *Bi — II —* '28.

T	p in mm Hg	R/R_0	i in mA	Datum
4.206	765	0.1234	12	20 December 1928
2.996	164	0.1228	„	„
2.487	61	0.1225	„	„
2.056	20.8	0.1224	„	„
1.430	3.3	0.1221	„	„
1.29	1.6	0.1220	„	„

Fig. 1.

§ 3. *Diskussion.*

Die folgenden Möglichkeiten bestehen noch:

1°. Entweder Wismuth oder Gold ist bei sehr niedrigen Temperaturen supraleitend. Bei 1°.25 K., wenn die eutektische Mischung sich schon erheblich unter ihrem Sprungpunkte befindet, sind jedoch weder Gold noch Wismuth supraleitend.

2°. Verunreinigungen können eine Erscheinung von Suprakonduktivität vortauschen, während diese in Wirklichkeit nicht auftritt.

Nur mit vollkommen reinen Komponenten kann diese Frage gelöst werden und Vorbereitungen sind deshalb schon getroffen das Wismuth noch weiter zu reinigen bis es spektroskopisch auch von den zuletzt verschwindenden Linien frei sein wird.

Sollten Veruntreinigungen eine Rolle spielen, so wäre es jedenfalls interessant wie eine so winzige Menge im Stande wäre Suprakonduktivität hervorzurufen.

Es läge dann vielleicht auf der Hand zu denken an eine Ausscheidung konzentrierter Verunreinigungen an der Oberfläche der das Eutektikum bildenden Kristalle.

Gegen die Hypothese, dass die Suprakonduktivität Verunreinigungen zu verdanken wäre, spricht die Tatsache, dass die Sprungpunkt-temperatur der eutektischen Mischung mit keiner der bekannten Sprungpunkt-temperaturen von Suprakonduktoren übereinstimmt.

Ausserdem hat die Temperatur-Widerstand Kurve den nämlichen Charakter als die eines wirklichen Suprakonduktors.

Unsere Erfahrung über das Verhalten von Suprakonduktoren hat uns gelehrt, dass winzige Beimischungen eines Suprakonduktors ein Metall, das in der Nähe der Gruppen von Suprakonduktoren steht, supraleitend machen können.

Die Abnahme des Widerstandes streckt sich dann über ein Temperaturintervall von einigen Graden aus, während bei "klassischen" Suprakonduktoren der "Fall" sich innerhalb $^1/_{20}{}^\circ$ vollzieht.

Der „Fall" der eutektischen Mischung jedoch ist „klassisch" und sehr scharf, während eine kontinuïrliche konzentrierte Beimischung eines Suprakonduktors, welche den jähen Fall verursachen könnte, dies doch wahrscheinlich bei seiner eigenen kennzeichnenden Sprungpunkt-temperatur tun würde.

Hingegen würde das verschiedene Verhalten (sehr ungleiche Restwiederstände beim Widerstandsfall und wenig ungleiche Sprungpunkte) von *Au-Bi*-I (in Gent hergestellt) und von *Au-Bi*-II (in Leiden hergestellt) wieder auf einen grossen Einfluss von Verunreinigungen hinweisen.

Wie dies auch sein möge, wir wünschen die obenstehende Mitteilung als eine vorlaüfige zu betrachten und werden erst dann mit absoluter Sicherkeit sprechen können, wenn die Messungen mit spektrographisch reinen Komponenten wiederholt worden sind.

Mathematics. — *A Representation of the Quadruple Set of the Biquadratic Twisted Curves of the First Kind that pass through Six given Points, on the Points of a Linear Four-dimensional Space.* By J. W. A. VAN KOL. (Communicated by Prof. HENDRIK DE VRIES).

(Communicated at the meeting of November 24, 1928).

§ 1. In the following way we produce a representation of the biquadratic twisted curves k^4 of the first kind lying in a linear three-dimensional space R_3 that pass through six given points $A_1, \ldots, A_6$, on the points of a linear four-dimensional space R_4. We choose two more points B_1 and B_2 in R_3. Any curve k^4 may be considered as the curve of intersection of a quadratic surface $\omega^2{}_1$ that passes through $A_1, \ldots, A_6$ and B_1 and another quadratic surface $\omega^2{}_2$ that passes through $A_1, \ldots, A_6$ and B_2. In R_4 we choose two arbitrary straight lines l_1 and l_2. We suppose the net of the quadratic surfaces $\omega_i{}^2$ to be projectively represented on the system of the ∞^2 planes λ_i $(i=1, 2)$ through l_i. To k^4 we associate as image the point of intersection of the planes λ_1 and λ_2 corresponding to $\omega_1{}^2$ and $\omega_2{}^2$. Inversely an arbitrary point in R_4 is the image of one curve k^4.

§ 2. *l_1 and l_2 are cardinal lines; an arbitrary point P of l_i is the image of the system of the ∞^2 curves k^4 that lie on the surface $\omega_k{}^2$ which is associated to the plane $P\,l_k$; the system of the ∞^3 curves k^4 represented on l_i lies on a pencil β_k of surfaces $\omega_k{}^2$ $(i, k=1, 2;\ i \neq k)$.*

These two nets of quadratic surfaces have a pencil of quadratic surfaces $\omega^2{}_{12}$ in common. Any surface $\omega^2{}_{12}$ contains ∞^2 curves k^4. To the pencil of surfaces $\omega^2{}_{12}$ that is a pencil of surfaces $\omega_1{}^2$ as well as a pencil of surfaces $\omega_2{}^2$, there correspond in R_4 two pencils of planes with axes l_1 and l_2 which are projective to each other. Consequently the system of the ∞^3 curves k^4 lying on the surfaces $\omega^2{}_{12}$ is represented on a conic k^2 that cuts l_1 and l_2.

Accordingly there is a cardinal conic k^2 that cuts l_1 and l_2; in any point of k^2 a system of ∞^2 curves k^4 is represented which lies on a quadratic surface $\omega^2{}_{12}$ that passes through $A_1, \ldots, A_6$, B_1 and B_2.

§ 3. The ∞^2 curves k^4 that are found by cutting an individual of β_1 by one of β_2, are singular for the representation; for any such a curve has ∞^1 image points, viz. all the points of a transversal of l_1 and l_2.

Also the ∞^2 curves k^4 that pass through B_1 or B_2, are singular for the representation. Any curve k^4 through B_1 or B_2 lies on a surface $\omega^2{}_{12}$. Hence any curve k^4 through B_i has all the points of a transversal of k^2 and l_k as image points.

The curve k^4 that passes through B_1 as well as through B_2, has ∞^2 image points, viz. all the points of the plane of k^2.

§ 4. The system Σ_1 of the curves k^4 that pass through a given point P, is represented on a plane α_P, viz. the plane of intersection of the linear spaces associated to the pencils of surfaces $\omega^2{}_1$ and $\omega^2{}_2$ that pass through P. α_P cuts l_1, l_2 and k^2.

The point of intersection of two planes α_P and α_Q is the image of the curve k^4 that passes through P as well as through Q.

§ 5. Let O_b be the image surface of the system Σ_2 of the curves k^4 that have a given chord b. There is one curve Σ_2 that lies on an individual of a pencil of surfaces $\omega_1{}^2$ as well as on an individual of a pencil of surfaces $\omega^2{}_2$. For these pencils produce two quadratic point-involutions on b that have one common pair of points. A plane that cuts l_1 and l_2 has, accordingly, one point outside l_1 and l_2 in common with O_b. As any surface $\omega^2{}_1$ or $\omega_2{}^2$ contains one curve of Σ_2, l_1 and l_2 are single lines of O_b. Consequently a plane cutting l_1 and l_2 has in all three points in common with O_b. O_b is, therefore, a cubic surface. From the above it also follows that k^2 lies on O_b.

α_P has no point outside l_1, l_2 and k^2 in common with O_b, according to the property that as a rule there is no biquadratic curve of the first kind that passes through seven given points and has a given chord. This property follows directly from the fact that the curves of Σ_1 lie on the quadratic surface defined by $A_1, \ldots, A_6$ and b.

From this it follows that there is one curve k^4 that has two given chords b_1 and b_2. Hence the surface O_{b_1} and O_{b_2} can only have one point in common outside l_1, l_2 and k^2. We can prove this by considering O_{b_1} as well as O_{b_2} as the rest of the intersection of two quadratic spaces that have a plane in common.

§ 6. Let Ω_l be the image space of the system Σ_3 of the curves k^4 that cut a given line l. We determine the degree of Ω_l through intersection with an arbitrary line a. a is the image of a system of ∞^1 curves k^4 produced through a projective correspondence between a pencil of surfaces $\omega_1{}^2$ and a pencil of surfaces $\omega^2{}_2$. This projectivity produces a (2,2)-correspondence on l that has four coincidences, which means that a cuts Ω_l in four points. Consequently Ω_l is a biquadratic space. l_1 and l_2 are double lines and k^2 is a double conic in Ω_l. We prove this by means of the intersection of Ω_l and a line that cuts l_1, l_2 or k^2.

§ 7. Two spaces Ω_l and Ω_m have a surface O_{lm} of the degree sixteen in common, which is the image of the system Σ_4 of the curves k^4 that cut two given lines l and m. l_1 and l_2 are quadruple lines and k^2 is a quadruple conic of O_{lm}.

α_P cuts O_{lm} outside l_1, l_2 and k^2 in four points.

Accordingly there are four curves k^4 that pass through a given point P and cut two given lines l and m [1]).

§ 8. Besides l_1, l_2 and k^2 the surface O_{lm} and the space Ω_n have a curve k_{lmn} of the order 32 in common, which is the image of the system Σ_5 of the curves k^4 that cut three given lines l, m and n.

The number of points of intersection of k_{lmn} and l_1 is equal to the number of curves of Σ_5 that lie on individuals of β_2 and the number of points of intersection outside l_1 of k_{lmn} and a linear space through l_1 is equal to the number of curves of Σ_5 that lie on individuals of β_1. As these numbers are obviously equal to each other k_{lmn} cuts l_1 as well as l_2 in 16 points. Also the number of points of intersection of k_{lmn} and k^2 is equal to 16.

k_{lmn} and Ω_o have 32 points outside l_1, l_2 and k^2 in common.

Hence:

There are 32 curves k^4 that cut four given lines.

§ 9. Let Ω_φ be the image space of the system Σ_6 of the curves k^4 that touch a given plane φ. In order to determine its degree we cut Ω_φ by an arbitrary line a, which is the image of a system of curves k^4 produced through a projective correspondence between a pencil of surfaces $\omega^2{}_1$ and a pencil of surfaces $\omega^2{}_2$. Intersection with φ yields a pencil of conics $k_1{}^2$ and a pencil of conics $k_2{}^2$ that are in projective correspondence. We must now determine the number of conics $k_1{}^2$ that touch the associated conic $k_2{}^2$. Any conic $k_1{}^2$ is touched by six conics $k_2{}^2$. If now to any conic $k_1{}^2$ we associate the six conics that belong to the same pencil and through the said correspondence are associated to the six conics $k_2{}^2$ that touch $k_1{}^2$, there arises in this pencil a (6,6)-correspondence with twelve coincidences. This means that twelve conics $k_1{}^2$ touch the associated conics $k_2{}^2$. Accordingly Ω_φ is of the degree twelve. We can further prove that in Ω_φ l_1 and l_2 are sextuple lines and k^2 is a sextuple conic.

k_{lmn} and Ω_φ have 96 points in common outside l_1, l_2 and k^2.

Consequently there are 96 curves k^4 that cut three given straight lines and touch a given plane.

§ 10. It is also possible to examine the representations of different other systems of curves k^4, e.g. the systems of the curves k^4 that cut a

[1]) Cf. Prof. JAN DE VRIES, Eine Kongruenz von Raumkurven vierter Ordnung erster Art. Nieuw Archief v. Wisk., **15**, 229.

given line once and another given line twice, that touch two or three given planes, that touch a given plane and cut one or two given lines and others.

Together with the numbers already found, in the following table a few numbers that we can derive in this way will be united:

$P^7B = 0$	$P^7\nu^2 = 4$	$P^6B\nu^2 = 4$	$P^6\nu^4 = 32$
$P^6B^2 = 1$	$P^7\nu\varrho = 12$	$P^6B\nu\varrho = 12$	$P^6\nu^3\varrho = 96$
	$P^7\varrho^2 = 36$	$P^6B\varrho^2 = 36$	$P^6\nu^2\varrho^2 = 288$
			$P^6\nu\varrho^3 = 864$
			$P^6\varrho^4 = 2592$

Here P indicates the condition that a biquadratic twisted curve of the first kind passes through a given point, B that it cuts a given line twice, ν that it cuts a given line once, and ϱ that it touches a given plane.

§ 11. The above enables us to indicate properties of surfaces that are formed by systems of ∞^1 curves k^4.

The curves k^4 that cut three given lines l, m and n, form a surface of the degree 32 that has 16-fold points in $A_1, \ldots, A_6$ and on which l, m and n are quadruple lines,

The curves k^4 that cut two given lines l and m and touch a given plane φ, form a surface of the degree 96 that has 48-fold points in $A_1, \ldots, A_6$ and on which l and m are twelve-fold lines. Etc.

We can prove e.g. that the former surface has 16-fold points in $A_1, \ldots, A_6$ by determining the number of points of intersection outside A_i of this surface and a straight line through A_i, which number is equal to the number of points outside l_1, l_2 and k^2 in which k_{lmn} cuts the image space of the system of the curves k^4 that cut a line through A_i outside A_i. By means of the method indicated in § 6 we can prove that this image space is quadratic and contains l_1, l_2 and k^2.